OPTICAL DESIGN

AND

LENS COMPUTATION

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BY

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PREFACE

THE designing and computation of lens systems and optical components is a subject of fundamental importance for the production of high grade scientific apparatus whether it be used for commercial or research purposes.

It has frequently been looked upon as a difficult subject and one which could only be carried out by those who had exceptional mathematical ability. In recent years, however, this adverse outlook has been largely removed and the subject has now been made available to a much wider field of workers.

This was primarily due to the late Professor A. E. Conrady and to the instructional work given on this subject in the Technical Optics Department of the Imperial College, London. Conrady in his book *Applied Optics and Optical Design* (Oxford University Press, England, 1929, Reprinted in America, 1943) set out the whole subject in a most lucid manner and described the methods developed by him for dealing with and correcting the aberrations which occur when optical systems are employed for producing images of high quality and definition.

Various other optical designers such as Fraunhofer, Steinheil, von Rohr, Dennis Taylor, Chrétien, T. Smith, Haselkuss, Lee, Booth and others, have developed their own highly successful methods in designing; but probably no one man made the processes of lens design so readily understandable to a larger number of people than did Conrady.

These pages deal with the practical application of the theory given by Conrady and are the result of the writer's experience in teaching optical computation and design extending over many years.

Numerical examples have been worked out in detail to illustrate the systematic way in which the design of most of the well-known forms of optical systems may be attempted. This does not mean that the book consists merely of columns of figures (for indeed the majority of the computations are omitted) but essential examples are given in order to illustrate the trend of a design and to show the way in which particular calculations are best carried out.

The author would like to stress, however, the importance of carrying out the numerical computations *completely* for any one design, for in so doing many important points may often be revealed which could not be predicted from purely theoretical considerations, and consequently a wider view of the designing problem as a whole is frequently secured.

Such numerical work has quite obviously taken a very considerable time and the size of the book is no indication of the man-hours put into the various computations.

It should be pointed out, also, that none of the worked-out designs given here are intended as representing a finality in design, but they are given as

indications as to the way in which particular problems may be approached; the reader may then gain for himself by further work the "optical instinct" which begins to develop by experience in the finishing stages of a design.

This book was also written because of the requests of numbers of individuals seeking the practical application of design theory. The author has always felt a little nervous about answering these requests because of the possibility of errors creeping in; but although the writer has worked out every example personally, he has had the additional and unique assistance of numbers of post-graduate students who have also been through these computations from time to time, and therefore the chances of errors have been greatly lessened. Although these students shall be nameless, the author would like to express his appreciation to them for their indirect help in the compilation of this work.

In no less a measure would he express his appreciation to the publishers for attempting such a difficult and arduous task of reproducing a book of this kind.

It is hoped that these pages may prove to be of some value and help to those who are anxious to obtain some information concerning the practical side of a seemingly difficult subject; but whilst the author has spent considerable time and patience in preparing this work, he feels that the real credit should go to the late Professor A. E. Conrady for without his theoretical methods this book could not have appeared.

In conclusion, I wish to express my appreciation of Professor L. C. Martin's valuable assistance during the proof-reading.

B. K. Johnson

Royal College of Science, London
1948

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Fundamental Formulæ for Ray-tracing Methods

THE fundamental ray-tracing formulæ are based strictly on the laws of refraction, and may be derived quite simply by reference to Fig. 1. For an incident ray OP on the spherical surface PA, the initial data would be L (the length AB from the pole of the surface A), r (the radius of curvature of the surface) and U (the angle under which the initial ray OP would make with the axis if produced to B). It is required to find the intersection length L' of the refracted ray PB′ and the angle U' which this ray makes with the axis. In the triangle CBE :

$$\frac{\mathrm{CE}}{L-r} = \sin U \text{ and } \therefore \mathrm{CE} = (L-r)\sin U$$

Similarly
$$\frac{\mathrm{CE}}{r} = \sin I$$

Hence
$$(L-r)\sin U = r \,.\, \sin I$$

$$\therefore \sin I = \sin U \,.\, \frac{(L-r)}{r} \qquad . \quad . \quad . \quad . \quad . \qquad (1)$$

From 2nd law of refraction,

$$N \,.\, \sin I = N' \,.\, \sin I' \,; \quad \therefore \sin I' = \sin I \,.\, \frac{N}{N'} \qquad . \quad . \quad . \qquad (2)$$

From the diagram

$$I + U = I' + U', \quad \therefore U' = U + I - I' \qquad . \quad . \quad . \qquad (3)$$

Also from the diagram

$$\frac{L'-r}{r} = \frac{\sin I'}{\sin U'}, \quad \therefore L'-r = \sin I' \,.\, \frac{r}{\sin U'} \qquad . \quad . \quad . \qquad (4)$$

And

$$L' = (L'-r) + r \qquad . \quad . \quad . \quad . \quad . \qquad (5)$$

In order to use the foregoing formulæ, it is necessary to adopt the following sign convention :—

Lengths

Lengths to the right of A (Fig. 1) are positive.
Lengths to the left of A are negative.

Angles

For rays meeting the axis, measure from *axis to ray*; then a clockwise direction indicates a positive angle and an anticlockwise direction indicates a negative angle.

For rays meeting a surface *away* from the axis (e.g., the ray OP in figure 1) measure from *ray to radius*; then a clockwise direction indicates a positive angle and an anticlockwise direction indicates a negative angle.

Lettering

Letters to the *left* of the surface are *plain*.
Letters to the *right* of the surface (if real) are *dashed*.

Items for aiding the Calculations

For optical ray-tracing, six-figure logarithms and trigonometrical functions will in general give the required accuracy. In telescope design five-figure work may be sufficient whereas in systems such as photographic lenses and microscope objectives, six or more figure accuracy may be required. In the

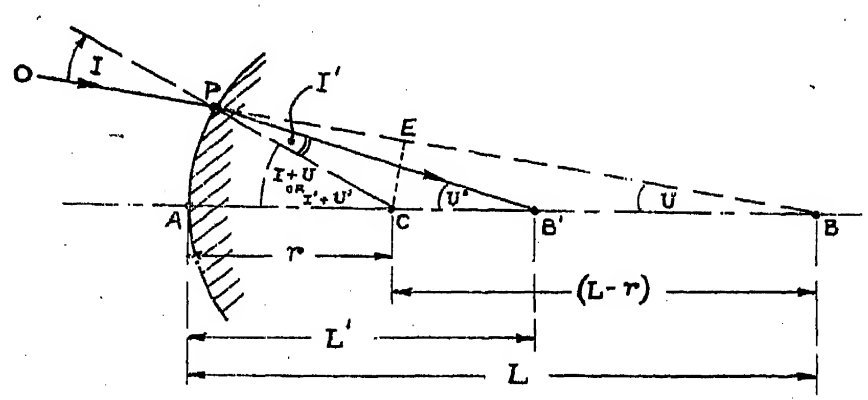


Fig. 1.

former case Bremiker's five-figure tables (published by H.M. Stationery Office) will be found useful, whilst in the latter case such tables as Bremiker's six-figure logarithms and trigonometrical functions, Peters' seven-figure trigonometrical functions only, and Bruhns' seven-figure logarithms and trigonometrical functions, will serve well. Bremiker's six-figure tables have proved particularly useful; moreover this book is exceptionally free from errors. The author has used practically every page of these tables during many years of computing work and has yet to find a mistake. Further, the section which gives log sines and log tangents directly to one second of arc up to five degrees is very helpful in certain calculations; for this saves interpolation which seems to be the chief cause of mistakes throughout any computation. Barlow's tables of reciprocals, squares, cubes, etc., are also valuable.

The use of calculating machines should not, of course, be overlooked as this may speed up the numerical work involved in ray-tracing. But it must be remembered that the trigonometrical functions must still be turned up from a book and then inserted in the machine before the latter can again be brought into operation ; and it may be that the total time taken for the complete operation will be longer than by using tables alone. This question of time taken is still a debatable point, as it depends much on the individual, but as it is often desirable to have a record of the actual angles of the rays in passing from one surface to another throughout the lens system, the use of tables giving logarithms and trigonometrical functions appears to lend itself more suitably to this work for those who are commencing ray-tracing. This remark does not apply, of course, to the advanced computer who may use his own methods involving various forms of machines and/or tables.

We will, therefore, confine ourselves in this book to the use of tables for the computations.

There are one or two minor points which should be taken note of before proceeding with an actual ray-trace. One of these is the decision as to what to do with terminal fives. For example, if we have a logarithm such as 0·12345(5) and we are working to five significant figures, the general rule is to round off the fifth figure to the nearest *even* number, so that the above log would now be 0·12346. Whereas if the log was 0·12344(5), it would now read 0·12344.

Secondly, when taking out a log of a *negative* number this should be indicated in some way in the computing columns preferably by a small letter n (e.g., the log of $-1·23456$ would be $0·09151\,n$) and should there be two logarithms together both with suffixes n, for example when multiplying two natural numbers, the resultant logarithm will be *without* an n.

These n's must be carefully watched throughout the calculation in order to obtain the correct sign to the result of the complete computation.

Thirdly, in order to lessen the possibility of errors it has been found convenient to use *positive* characteristics for all logarithms, for then they may always be added and not both added and subtracted, viz. :—

Natural number	*Ordinary method of using characteristics*	*Optical computer's method of using characteristics*
12·3456	1·	1·
1·23456	0·	0·
0·12345	$\bar{1}$·	9·
0·01234	$\bar{2}$·	8·
0·00123	$\bar{3}$·	7·

If we take the following simple calculation, the advantage will be evident :—

$$17 \cdot 23 \times 0 \cdot 0072 \times 0 \cdot 673$$

	Usual method	*Optical computer's method*
log 17·23	1·23628	1·23628
+ log 0·0072	$\bar{3}$·85733	7·85733
+ log 0·673	$\bar{1}$·82802	9·82802
log sum	$\bar{2}$·92163	8·92163

The use of cologs also helps in a similar manner. The colog of a number is the log of the reciprocal of that number (i.e., colog $N = \log\left(\frac{1}{N}\right) = \log 1 - \log N$). If we take log 1 as being equal to 9·9999(10) then we can subtract in the head log N from this, and write the result straight down. In the previous example let us imagine the numerator to be divided by a denominator of 0·0062 × 7·35, then

$$\text{colog } 0 \cdot 0062 = \log 1 - \log 0 \cdot 0062 = 9 \cdot 9999(10) - 7 \cdot 79239$$
$$= 2 \cdot 20761$$

and similarly colog 7·35 = 9·13371.

So that the optical computer's method of doing this simple calculation, namely $\dfrac{17 \cdot 23 \times 0 \cdot 0072 \times 0 \cdot 673}{0 \cdot 0062 \times 7 \cdot 35}$ would be :—

log 17·23	= 1·23628
+ log 0·0072	= 7·85733
+ log 0·673	= 9·82802
+ colog 0·0062	= 2·20761
+ colog 7·35	= 9·13371
log sum	= 0·26295

where all the logs and their characteristics can simply be added.

An Initial Illustrative Exercise

In order to become acquainted with the ray-tracing process let us take the following example :—

A ray is to be traced at an inclination of five degrees from an object situated at 24 inches from the front surface of a bi-convex lens of refractive index 1·5180 and having radii equal to forty inches and eight inches for the

first and second surfaces respectively (see Fig. 2). The axial thickness of the lens is to be $0 \cdot 60$ inches. We wish to determine the intersection length L' of this ray after it has passed through the lens.

The initial data will be therefore :—

$L = -24 \cdot 000$ inches $\qquad r_1 = +40 \cdot 000$ inches
$U = -5°–0'–0''$ $\qquad r_2 = -\ 8 \cdot 000$ inches.
Axial thickness $d_1' = 0 \cdot 600$ inches.
Refractive index of glass $= 1 \cdot 5180$.

A column is ruled for each surface and the quantities in the fundamental formulæ (given on page 1) are written down on the left hand side. The calculation (No. 1) should be followed carefully ; the logs turned up, attention

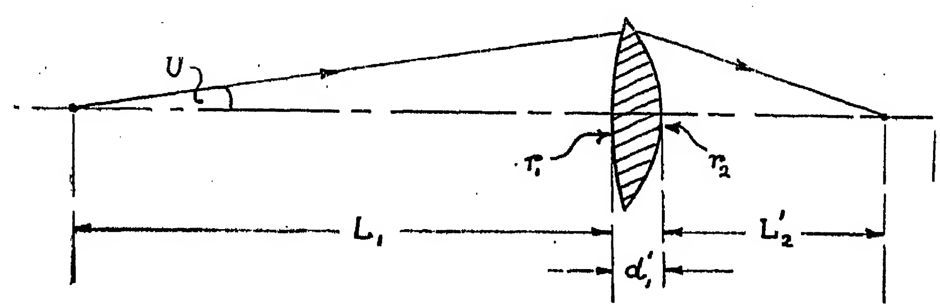


Fig. 2.

being given particularly to signs, and the arithmetical work checked. And then one may carry out the calculation for oneself on a separate piece of paper without referring to the book. It is nearly always desirable to make a diagrammatic sketch of the problem in hand immediately above the computing columns, so that a mental picture may be formed of the direction of the rays as they pass through the surfaces. This has often been the means of checking whether the correct sign has been given to an angle in the angle register of the computing columns. Five significant figures in the logs will give the necessary accuracy for ray-tracing work, with the exception of paraxial angles which are taken to six places. The radius of a surface cannot in practice be produced to a greater accuracy than one thousandth of an inch and therefore radii are only given to one in the third place of decimals.

Paraxial Formulæ

If an angle U is expressed in radians or circular measure, then

$$\sin U = U - \frac{1}{6} U^3 + \frac{1}{120} U^5 \quad . \quad . \quad . \quad . \quad . \quad \text{etc.}$$

so that if the angle U is small, the higher terms of this series become almost negligible and consequently $\sin U$ may be taken as being equal to U itself.

In optical calculations the required accuracy in the ray direction is of the order of one second of arc or $0 \cdot 0000048$ radians, so that utilizing the

CALCULATION No. 1

	First surface	*Second surface*
L	-24.000	-54.0006
$-r$	-40.000	$+\ 8\cdot000$
$(L-r)$	$-64\cdot000$	$-46\cdot0006$
$\log\sin U$	$8\cdot94030\ n$	$8\cdot59486\ n$
$+\log(L-r)$	$1\cdot80618\ n$	$1\cdot66276\ n$
$\log(L-r)\sin U$	$0\cdot74648$	$0\cdot25762$
$-\log r$	$1\cdot60206$	$0\cdot90309\ n$
$\log\sin I$	$9\cdot14442$	$9\cdot35453\ n$
$+\log\left(\frac{N}{N'}\right)$	$9\cdot81873$	$0\cdot18127$
$\log\sin I'$	$8\cdot96315$	$9\cdot53580\ n$
$+\log r$	$1\cdot60206$	$0\cdot90309\ n$
$\log r\,.\sin I'$	$0\cdot56521$	$0\cdot43889$
$-\log\sin U'$	$8\cdot59486\ n$	$8\cdot91850$
$\log(L'-r)$	$1\cdot97035\ n$	$1\cdot52039$
	Angle Register	
U	$-5°-\ 0'-\ 0''$	$-\ 2°-15'-17''$
$+I$	$8°-\ 0'-58''$	$-13°-\ 4'-29''$
$U+I$	$3°-\ 0'-58''$	$-15°-19'-46''$
$-I'$	$5°-16'-15''$	$+20°-\ 5'-\ 3''$
U'	$-2°-15'-17''$	$4°-45'-17''$
$(L'-r)$	$-93\cdot4006$	$33\cdot1428$
$+r$	$40\cdot000$	$-\ 8\cdot000$
L'	$-53\cdot4006$	$25\cdot1428$
$-d'$	$0\cdot600$	
new L	$-54\cdot0006$	

second approximation only for the value of a sine (namely $\sin U = U - \frac{1}{6}U^3$) we may treat $U = \sin U$ up to the value of U which makes $\frac{1}{6}U^3 = 0\cdot0000048$ or $U = 0\cdot031$ = nearly two degrees.

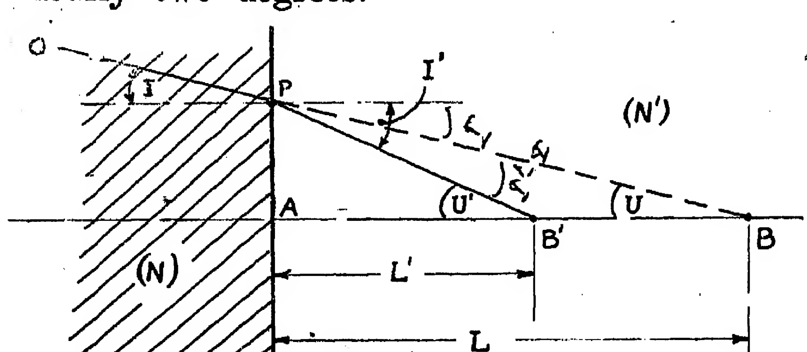


Fig. 3.

Therefore, the sines in the fundamental formulæ may be replaced by the angles themselves for paraxial angles. Denoting these by small letters, the paraxial formulæ then become as shown in the second column below :—

	Marginal Ray Formulæ	*Paraxial Ray Formulæ*
(1)	$\sin I = \sin U \dfrac{(L-r)}{r}$	$i = u . \dfrac{l-r}{r}$
(2)	$\sin I' = \sin I . \dfrac{N}{N'}$	$i' = i . \dfrac{N}{N'}$
(3)	$U' = U + I - I'$	$u' = u + i - i'$
(4)	$L' - r = \sin I' . \dfrac{r}{\sin U'}$	$l' - r = i' . r/u'$
(5)	$L' = (L' - r) + r$	$l' = (l' - r) + r.$

Marginal and Paraxial Ray-Tracing

The tracing of rays at a number of finite angles is not the most usual type of optical calculation. As a rule, a good judgment of the state of correction of a lens or lens system can be based on the tracing of only two rays, one through the marginal part of the lens and the other through the axial (or as it is sometimes called) the paraxial region. Nevertheless, it is advisable in certain cases to trace one zonal ray.

We will, therefore, trace a paraxial ray through the lens whose specification is given on page 5, and assuming that the initial angle U of five degrees represents the extreme marginal ray, we shall then be able to determine the spherical aberration $(l' - L')$.

Paraxial angles. The linear character of the computing formulæ causes them to give precisely the same intersection lengths no matter what value may be given to the first u, and therefore the latter may be arbitrarily chosen.

The paraxial angle u at the first surface is taken as a nominal value which is precisely equal to the first $\sin U$. Its value is the natural number given in the log tables to the log $\sin U$. Subsequent values of the paraxial angles are the natural numbers of their logs given in the paraxial column.

An example of the paraxial ray-trace is shown in calculation No. 2.

Formulæ for use with Plane Surfaces

Referring to Fig. 3, an incident ray OP (meeting the axis at B) is refracted at a glass-air surface PA and cuts the axis at B′.

From the diagram $\hat{I} = \hat{U}$ and $\hat{I}' = \hat{U}'$ . . . . . . Pl. (1)

CALCULATION No. 2

	First surface	*Second surface*
l $-r$	$-24 \cdot 000$ $-40 \cdot 000$	$-53 \cdot 4582$ $+8 \cdot 000$
$(l-r)$	$-64 \cdot 000$	$-45 \cdot 4582$
$\log u$ $+\log(l-r)$	$8 \cdot 94030\ n$ $1 \cdot 80618\ n$	$8 \cdot 59739\ n$ $1 \cdot 65761\ n$
$\log(l-r)u$ $-\log r$	$0 \cdot 74648$ $1 \cdot 60206$	$0 \cdot 25500$ $0 \cdot 90309\ n$
$\log i$ $+\log\left(\frac{N}{N'}\right)$	$9 \cdot 14442$ $9 \cdot 81873$	$9 \cdot 35191\ n$ $0 \cdot 18127$
$\log i'$ $+\log r$	$8 \cdot 96315$ $1 \cdot 60206$	$9 \cdot 53318\ n$ $0 \cdot 90309\ n$
$\log r.i'$ $-\log u'$	$0 \cdot 56521$ $8 \cdot 59739\ n$	$0 \cdot 43627$ $8 \cdot 88595$
$\log(l'-r)$	$1 \cdot 96782\ n$	$1 \cdot 55032$
u $+i$	$-0 \cdot 087157$ $0 \cdot 139450$	$-0 \cdot 039572$ $-0 \cdot 224859$
$u+i$ $-i'$	$0 \cdot 052293$ $0 \cdot 091865$	$-0 \cdot 264431$ $0 \cdot 341335$
u'	$-0 \cdot 039572$	$0 \cdot 076904$
$(l'-r)$ $+r$	$-92 \cdot 8582$ $40 \cdot 000$	$35 \cdot 5075$ $-8 \cdot 000$
l' $-d'$	$-52 \cdot 8582$ $0 \cdot 600$	$27 \cdot 5075$
new l	$-53 \cdot 4582$	

Therefore, the Spherical Aberration $(l'-L')$

$$= 27 \cdot 5075 - 25 \cdot 1428 \text{ (from page 6)}$$
$$= +2 \cdot 3647 \text{ inches.}$$

By substituting $\hat{U}$ for $\hat{I}$ in standard equation (2), namely

$$\sin I' = \sin I \cdot \frac{N}{N'}$$

$$\sin U' = \sin U \cdot \frac{N}{N'} \quad . \quad . \quad . \quad . \quad . \quad \text{Pl. (2)}$$

Also from the diagram $\frac{L'}{\mathrm{PA}} = \frac{1}{\tan U'}$, and $\frac{L}{\mathrm{PA}} = \frac{1}{\tan U}$ and therefore

$$\frac{L'}{L} = \frac{1}{\tan U'} \cdot \tan U = \text{cotan } U' \cdot \tan U$$

hence $\qquad L' = L \cdot \tan U \cdot \text{cotan } U'.$

Putting sin/cos in place of the tangents, the equation now becomes

$$L' = L\left(\frac{\sin U}{\sin U'}\right)\left(\frac{\cos U'}{\cos U}\right)$$

And from Pl. (2) above $\frac{\sin U}{\sin U'} = \frac{N'}{N}$ so that

$$L' = L \cdot \frac{N'}{N} \cdot \frac{\cos U'}{\cos U} = L \cdot \frac{N'}{N} \cdot \frac{\sec U}{\sec U'} \quad . \quad . \quad \text{Pl. (3)}$$

Thus, the formulæ for ray-tracing through a plane surface are as follows :—

	Marginal rays	*Paraxial rays*
Pl. (1)	$I = U$ and $I' = U'$	$i = u$ and $i' = u'$
Pl. (2)	$\sin U' = \sin U \cdot \frac{N}{N'}$	$u' = u \cdot \frac{N}{N'}$
Pl. (3)	$L' = L \cdot \frac{N'}{N} \cdot \frac{\sec U}{\sec U'}$	$l' = l \cdot \frac{N'}{N}$

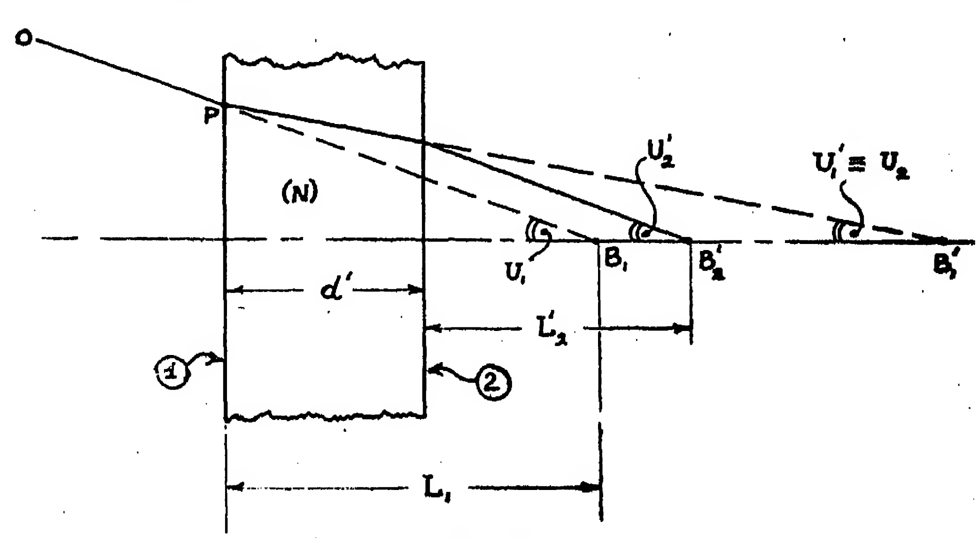


Fig. 4.

Let us make use of these formulæ by taking an example (see Fig. 3): a ray OP in a medium of refractive index $N = 1\cdot5180$ is travelling in the direction OPB with $L = 22\cdot700$ inches and $U = 3°–8'–9''$ and is refracted into a medium of refractive index N'. We wish to know the intersection length L' and the angle U' after refraction. A paraxial ray having $l = 23\cdot900$ inches and $u = 0\cdot051246$ will also be traced. The computing columns will be set out as in Calculation (3).

CALCULATION No. 3

⟶	*Marginal ray*	*Paraxial ray*	⟵
L U	22·700 3° – 8′ – 9″	23·900 0·051246	l u
$\log \sin U$ $+ \log\left(\frac{N}{N'}\right)$	8·73801 0·18127	8·70966 0·18127	$\log u$ $+ \log\left(\frac{N}{N'}\right)$
$\log \sin U'$	8·91928	8·89093	$\log u'$
U'	4° – 45′ – 48″	0·077791	u'
$\log \sec U$ $- \log \sec U'$	0·00065 −0·00150		
$\log. \frac{\sec U}{\sec U'}$ $+ \log\left(\frac{N'}{N}\right)$	9·99915 9·81873	 9·81873	 $\log\left(\frac{N'}{N}\right)$
$\log \frac{N'}{N} \cdot \frac{\sec U}{\sec U'}$ $+ \log L$	9·81788 1·35603	 1·37840	 $+ \log l$
$\log L'$	1·17391	1·19713	$\log l'$
L'	14·9249	15·7445	l'

Ray-tracing through a Plane Parallel Plate

Many cases arise in optical design when it is desired to know the aberrations caused by rays passing through a plane parallel plate of glass or its equivalent (e.g., a right-angle prism placed between any set of lenses may be treated as a parallel plate whose thickness is equal to the axial ray length through such a prism). Formulæ may be derived from those already deduced for one plane surface which will enable the ray-tracing to be carried out. Referring to Fig. 4 the following formulæ will be found valid :—

Marginal ray formulæ	*Paraxial ray formulæ*
$U_2' = U_1$	$u_2' = u_1$
$\sin U_1' = \dfrac{\sin U_1}{N}$	$u_1' = \dfrac{u_1}{N}$

Pl. (4) $L_2' = L_1 - d' \,.\, \tan U_1' \,.\, \text{cotan}\, U_1$ $\quad l_2' = l_1 - (d'/N)$

and the displacement

$$B_1B_2' = d' - d' \,.\, \left(\cos U_1 \,.\, \sec \frac{U_1'}{N}\right); \quad B_1B_2' = d' - (d'/N)$$

Putting these into practice, we may assume a ray OP from some lens system converging to a point B_1 under an angle U_1; it then passes through a parallel plate of thickness d' and after refraction at the second surface continues to B_2'. The distance L_2' is the intersection length required; and it is frequently useful to know the displacement B_1B_2'.

Assuming: $L_1 = 5 \cdot 000$ inches; $U_1 = 8°–42'–39''$
$d' = 1 \cdot 500$ inches*

and refractive index $N = 1 \cdot 5660$ (a glass frequently used for binocular prisms) then Calculation No. 4 will show the utilization of the above formulæ.

CALCULATION NO. 4

Marginal ray-trace

L_1	5·000
U_1	8° – 42′ – 39″
d'	1·500
log sin U_1 – log N	9·18026 0·19479
log sin U_1'	8·98547
U_1'	5° – 32′ – 59″
log tan U_1' + log cotan U_1 + log d'	8·98751 0·81470 0·17609
log 2nd term	9·97830
(–) 2nd term + L_1	– 0·95126 5·00000
L_2'	4·0487(4)

Determination of displacement $B_1 B_2'$

log sec U_1' – log N	0·00204 0·19479
log sec $\dfrac{U'}{N}$ + log cos U_1 + log d'	9·80725 9·99496 0·17609
log 2nd term	9·97830
(–) 2nd term + d'	– 0·95126 1·50000
$B_1 B_2'$	0·5487(4)

N.B.—Do not mistake *sine column* for *tangent column* when looking up the log tangents.

* Approximately the distance travelled by the axial ray through one right-angled prism when the latter is used in a prismatic binocular.

CALCULATION No. 5

First Surface ($r_1 = 3{\cdot}333''$)

Marginal Ray →		Paraxial Ray ←	
L	∞	∞	l
$-r$			$-r$
$(L-r)$			$(l-r)$
$\log \sin U$			$\log$ (nominal) u
	$Y = 0{\cdot}800$		
$+\log (L-r)$			$+\log (l-r)$
$\log (L-r) \sin U$	9·90309	same	$\log (l-r)u$
$-\log r$	0·52284	same	$-\log r$
$\log \sin I$	9·38025	same	$\log i$
$+\log \left(\frac{N}{N'}\right)$	9·81873	same	$+\log \frac{N}{N'}$
$\log \sin I'$	9·19898	same	$\log i'$
$+\log r$	0·52284	same	$+\log r$
$\log r.\sin I'$	9·72182	9·72182	$\log r \,.\, i'$
$-\log \sin U'$	8·92171	8·91330	$-\log u'$
$\log (L'-r)$	0·80011	0·80852	$\log(l'-r)$
U	0°– 0′– 0″	0·000000	u
$+I$	13°–53′–16″	0·240022	$+i$
$U+I$	13°–53′–16″	0·240022	$u+i$
$-I'$	9°– 5′–52″	0·158118	$-i'$
U'	4°–47′–24″	0·081904	u'
$L'-r$	6·3112	6·4346	$l'-r$
$+r$	3·333	3·333	$+r$
L'	9·6442	9·7676	l'
$-d'$	0·600	0·600	$-d'$
new L	9·0442	9·1676	new l

Second Surface ($r_2 = \infty$)

Marginal Ray		Paraxial Ray	
L	9·0442	9·1676	l
U	4°–47′–24″	0·081904	u
$\log \sin U$	8·92171	8·91330	$\log u$
$+\log \left(\frac{N}{N'}\right)$	0·18127	0·18127	$+\log \frac{N}{N'}$
$\log \sin U'$	9·10298	9·09457	$\log u'$
U'	7°–16′–57″	0·124328	u'
$\log \sec U$	0·00152		
$-\log \sec U'$	0·00352		
$\log \frac{\sec U}{\sec U'}$	9·99800		
$+\log \left(\frac{N'}{N}\right)$	9·81873	9·81873	$\log \frac{N'}{N}$
$\log \left(\frac{N'}{N}\right) . \frac{\sec U}{\sec U'}$	9·81673		
$+\log L$	0·95637	0·96226	$+\log l$
$\log L'$	0·77310	0·78099	$\log l'$
L'	5·9306	6·0393	l'

Spherical Aberration $(l' - L') = +\,0{\cdot}1087''$

Tracing Parallel Rays through a Lens or Lens System

There are many cases when optical instruments, such as telescopes, cameras, etc., have to be designed for use with an infinitely distant object point (i.e., with parallel light). Fig. 5a shows a plano-convex lens (with $r_1 = 3 \cdot 333$ inches, $r_2 = \infty$ (i.e., flat) and refractive index $= 1 \cdot 5180$) through which incident marginal and paraxial rays parallel to the axis, will be traced.

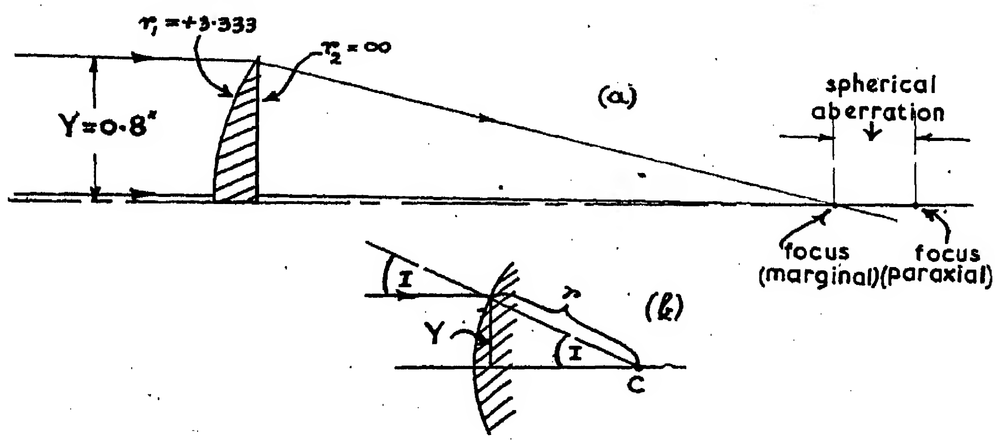


Fig. 5.

A slight modification has to be made in the fundamental opening equation $\sin I = (L - r) \sin U$; for $L = \infty$ and $U = 0$, and by referring to Fig. 5b it will be clear that $\sin I$ now becomes equivalent to Y/r where Y is the perpendicular height of the incident marginal ray from the optical axis; in this case taken as $0 \cdot 800$ inches.

By carrying out such a calculation (5) the spherical aberration $(l' - L')$ may be determined.

In the computing columns for the first surface, we write down $Y = 0 \cdot 800$ in place of the first five lines; and then put $\log (L - r) \sin U$ equal to the log of $0 \cdot 800$. The usual procedure is then continued.

Chromatic Aberration

It is as well, at this stage, to obtain some practice in determining trigonometrically the chromatic aberration of a lens, and we will take the preceding specification as a basis for this. For visual instruments it is usual to design the lens system so that the red part of the spectrum (namely, the C line $\lambda = 6563$A.) and the blue-green part (namely, the F line $\lambda = 4861$A.) are brought to a common focus, and therefore in carrying out this initial example we will trace two such rays through the lens. This necessitates a knowledge of the refractive indices for the C and F lines which are obtained from the optical constants given in a glass list. The latter shows that for a glass with $N_D = 1 \cdot 5180$ and a V value equal to $65 \cdot 1$, the partial dispersions are $0 \cdot 00250$ for C to D and $0 \cdot 00560$ for D to F; this gives therefore $N_C = 1 \cdot 5155$ and $N_F = 1 \cdot 5236$.

Further, when designing achromatic lenses a prescribed state of chromatic correction is best realized for the *whole* aperture of the lens system, if it is established trigonometrically for $\sqrt{0 \cdot 5}$ of the aperture, and here again, although we are dealing with an uncorrected lens, we will trace an initial "white" ray at $\sqrt{0 \cdot 5}$ of the semi-aperture (i.e., $0 \cdot 7071 \times 0 \cdot 8 = 0 \cdot 5657$) in order to determine the chromatic aberration for this zone (see Fig. 6).

Thus the data for this calculation (6) will be as follows :—

$r_1 = +3 \cdot 333$ inches Axial thickness $d' = 0 \cdot 600$ inches

$r_2 = \infty$ (i.e., flat)

Refractive indices $\begin{cases} N_C = 1 \cdot 5155 \\ N_F = 1 \cdot 5236 \end{cases}$

$Y = 0 \cdot 5657$ inches.

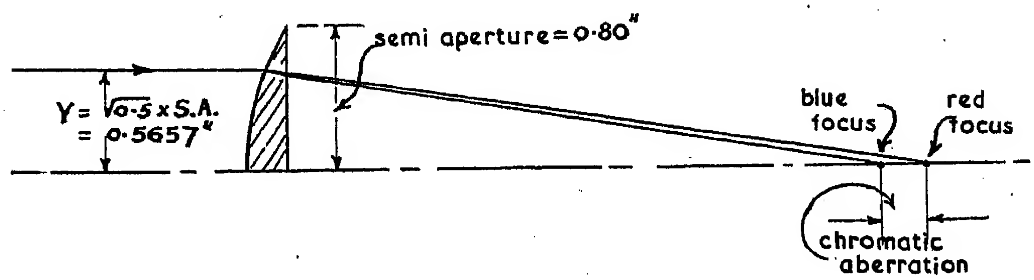


Fig. 6.

Method for Varying Spherical Aberration

One of the most useful means of changing the amount of spherical aberration present in a lens is by "bending" the lens, that is to say, by changing its shape without altering its power. The fact that the spherical aberration can be decreased or increased in this way is of fundamental importance in optical design and is applicable to simple lenses or complex lens systems in relation to any particular set of conditions of object and image distance.

The power F of a "thin" lens $= \dfrac{1}{f} = (N-1)\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$.

The term in the second bracket may be called the total curvature $\mathcal{R}$ of the lens, whilst the curvature of each surface can be expressed as $R_1 = \dfrac{1}{r_1}$ and $R_2 = \dfrac{1}{r_2}$.

Assuming therefore a total curvature $\mathcal{R} = 0 \cdot 300$ we can find any number of "bendings" of such a lens by adding the same amount algebraically to the curvature of each surface. For example, by putting $R_1 = -0 \cdot 10$ and substituting in the equation $\mathcal{R} = R_1 - R_2$, we get $R_2 = -0 \cdot 400$; and with respective radii $r_1 = -10 \cdot 000$ inches and $r_2 = -2 \cdot 500$ inches we find a shape of lens indicated in Fig. 7a. By choosing other values for the first radius, various shapes of the lens may be produced as illustrated in Fig. 7 (b) to (g), but always the same power or focal length of the lens is maintained.

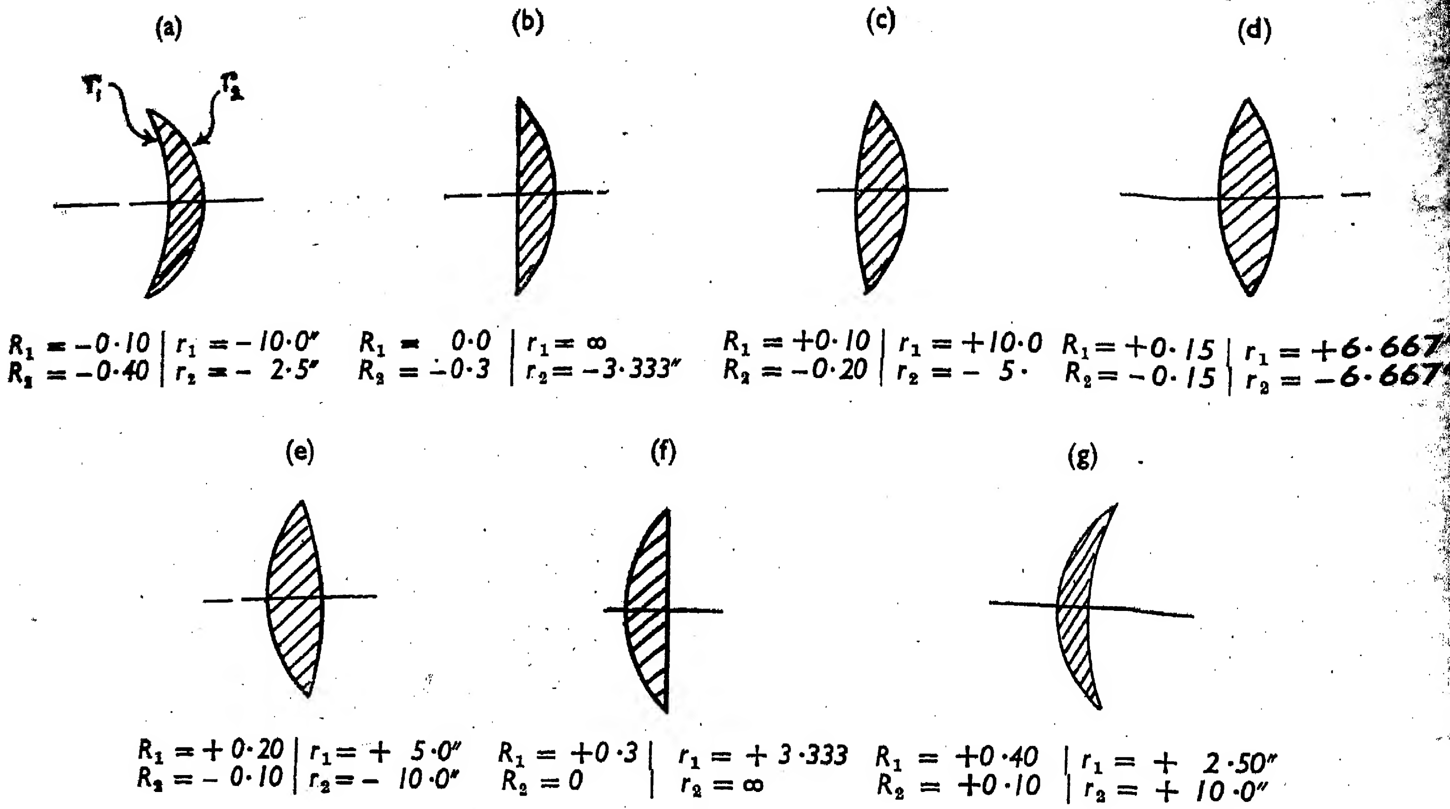


Fig. 7.

It is then instructive to determine the spherical aberration for these various shapes of the lens for a given set of conditions, and to plot the former on graph paper against the latter.

The determination of the spherical aberration for each lens shape can be carried out either by ray-tracing methods or by analytical methods (i.e., an algebraic approximation based on the assumption that the lens is infinitely thin).

The conditions stipulated here will be that the lens should have a focal length of 6·435 inches and refractive index 1·5180, which gives the previously mentioned total curvature of 0·300; and that the object distance should be 24 inches. If ray-tracing is to be used, which is excellent practice at this stage, it will be necessary further to fix the diameter of the lens in order to know the initial angle U of the marginal ray. Making the diameter 1·67 inches this would give two degrees as the initial marginal ray, and finally the axial thickness of the lens may be taken as 0·600 inches.

Tracing a marginal and paraxial ray through each shape of lens with the following data :—

$N = 1·5180$; r_1 and r_2 as specified in Fig. 7 (a) to (g)

$L = -24·000$ inches ; $U = -2°\text{–}0'\text{–}0''$; and $d' = 0·600$ inches

it will be found that the spherical aberration values are :—

Lens shape	*Spherical Aberration*
(a)	+ 1·1562 inches
(b)	+ 0·6391
(c)	+ 0·3194
(d)	+ 0·2417
(e)	+ 0·2154
(f)	+ 0·3111
(g)	+ 0·5608

Plotting these values as ordinate against the curvature R_1 of the first surface of the lens as abscissa, we get the parabola shown in Fig. 8 from which

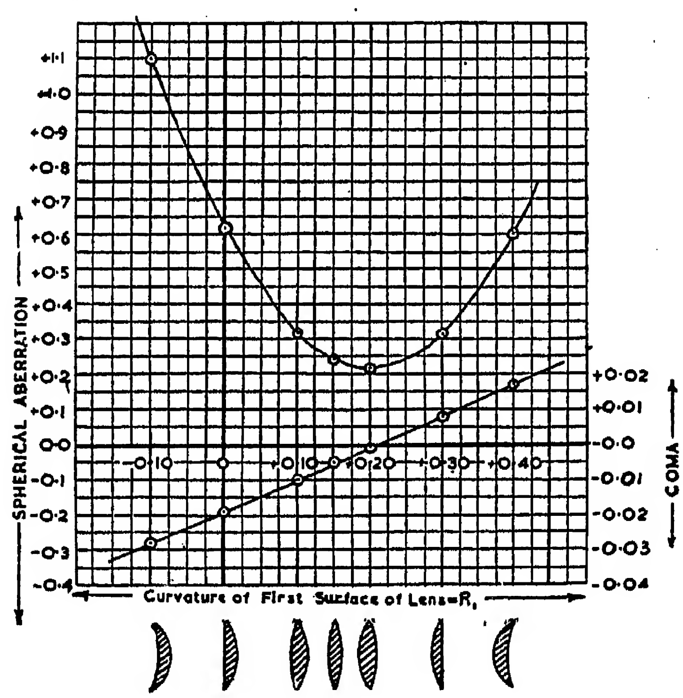


Fig. 8.

we see that the minimum amount of spherical aberration obtainable for the particular conditions set out above is $+0\cdot215$ inches, and this occurs when the lens has a curvature of the first surface $R_1 = +0\cdot210$, thus giving a radius $r_1 = +4\cdot762$ inches and $r_2 = -11\cdot111$ inches.

If the analytical method is used, the numerical work is much less and the time taken is greatly reduced, moreover for a simple problem of this kind the values obtained (whilst not being strictly accurate) agree fairly closely with the ray-tracing values.

The method to be used is one developed by Conrady and given in detail in his book *Applied Optics and Optical Design*, on pages 92 to

TABLE I

N	$\log G_1$	$\log G_2$	$\log G_3$	$\log G_4$	$\log G_5$	$\log G_6$	$\log G_7$	$\log G_8$
1·43	9·6431	9·9190	0·0559	9·7124	0·1648	9·9758	9·7637	9·4878
4	·6591	·9313	·0683	·7206	·1735	·9848	·7729	·5008
5	·6749	·9432	·0805	·7286	·1820	9·9936	·7819	·5136
6	·6904	·9550	·0925	·7365	·1904	0·0022	·7907	·5261
7	·7057	·9666	·1043	·7441	·1985	·0106	·7992	·5384
8	·7207	·9779	·1158	·7515	·2065	·0188	·8076	·5505
9	·7355	9·9890	·1272	·7588	·2142	·0269	·8159	·5624
1·50	9·7501	0·0000	0·1383	9·7659	0·2218	0·0348	9·8239	9·5740
1	·7645	·0108	·1493	·7729	·2293	·0425	·8318	·5855
2	·7787	·0214	·1600	·7797	·2366	·0500	·8395	·5968
3	·7926	·0318	·1707	·7863	·2437	·0574	·8471	·6079
4	·8064	·0420	·1811	·7928	·2507	·0647	·8545	·6189
5	·8200	·0521	·1914	·7992	·2576	·0718	·8618	·6297
6	·8334	·0621	·2015	·8055	·2643	·0788	·8689	·6403
7	·8466	·0718	·2115	·8116	·2709	·0857	·8759	·6507
8	·8597	·0815	·2213	·8176	·2774	·0924	·8828	·6611
9	·8726	·0910	·2310	·8235	·2838	·0990	·8896	·6712
1·60	9·8854	0·1004	0·2405	9·8293	0·2900	0·1055	9·8963	9·6812
1	·8980	·1096	·2500	·8350	·2962	·1119	·9028	·6911
2	·9104	·1187	·2593	·8406	·3022	·1182	·9092	·7009
3	·9227	·1277	·2684	·8460	·3081	·1243	·9155	·7105
4	·9348	·1366	·2775	·8514	·3140	·1304	·9217	·7200
5	·9468	·1454	·2864	·8567	·3197	·1364	·9279	·7294
6	·9587	·1540	·2952	·8619	·3253	·1423	·9339	·7386
7	·9705	·1625	·3039	·8670	·3309	·1480	·9398	·7478
8	·9821	·1710	·3125	·8720	·3364	·1537	·9457	·7568
9	9·9936	·1793	·3210	·8770	·3417	·1594	·9514	·7657
1·70	0·0050	0·1875	0·3294	9·8818	0·3470	0·1649	9·9571	9·7745
1	·0162	·1957	·3377	·8866	·3523	·1703	·9627	·7832
2	·0274	·2037	·3459	·8913	·3574	·1757	·9682	·7918
3	·0384	·2116	·3540	·8960	·3625	·1810	·9736	·8003
4	·0493	·2195	·3620	·9005	·3675	·1862	·9789	·8088
5	·0601	·2272	·3699	·9050	·3724	·1913	·9842	·8171
1·76	0·0708	0·2349	0·3777	9·9095	0·3772	0·1964	9·9894	9·8253

96. It gives the primary spherical aberration $LA'p$ contribution of a thin lens as :—

$$LA'p = LAp\left(\frac{l'}{l}\right)^2 + l'^2 \cdot y^2 \left[G_1 R^3 - G_2 R^2 R_1 + G_3 R^2 \frac{1}{l_1} + G_4 R R_1^2 - G_5 R R_1 \frac{1}{l_1} + G_6 R\left(\frac{1}{l_1}\right)^2\right]$$

where LAp represents any spherical aberration carried over from any previous lens, etc. ; should there be none this first term will automatically disappear.

l' is the image distance as calculated by the usual thin lens formula,

namely $\frac{1}{l'} = (N - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{l}$.

y the semi-aperture of the lens at which the aberration is required.

R the total curvature of the lens.

R_1 the curvature of the first surface.

l_1 the object distance

and the G values are functions of the refractive index N as defined on page 95 of Conrady's book. These have been calculated for a range of indices and a list of these log. values is given in Table I.

Collecting the various numerical values connected with our particular problem we have :—

$LAp\left(\frac{l'}{l}\right)^2 = 0$; $N = 1 \cdot 5180$; $l' = 8 \cdot 795$ inches

$y = 0 \cdot 8375$ inches ; $R = 0 \cdot 300$; $R_1 =$ unknown

$l_1 = -24 \cdot 000$ inches ; and G values from tables for $N = 1 \cdot 5180$.

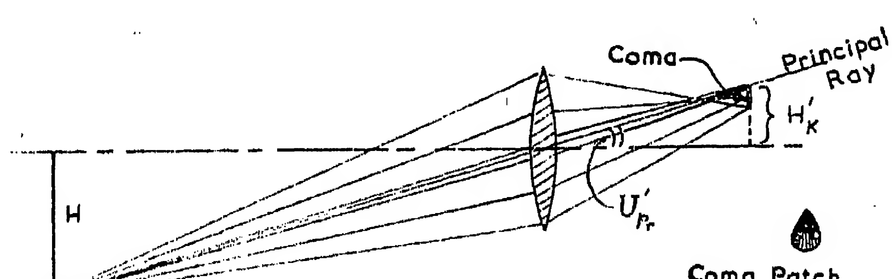


Fig. 9.

The Calculation (7) may be arranged as shown.

CALCULATION No. 7

$LA'p = y^2.(l')^2$	$\{ G_1\mathcal{R}^3$	$-G_2 . \mathcal{R}^2 R_1$	$+G_3\mathcal{R}^2 . \frac{1}{l_1}$	$+G_4 . \mathcal{R} . R_1^2$	$-G_5 . \mathcal{R}R_1\frac{1}{l_1}$	$+G_6 . \mathcal{R} . \left(\frac{1}{l_1}\right)^2 \}$
$\log G$	9·7759	0·0193 *n*	0·1579	9·7783	0·2351 *n*	0·0485
$+\log (l')^2$	1·8885	1·8885	1·8885	1·8885	1·8885	1·8885
$+\log \mathcal{R}^n$	8·4313	8·9542	8·9542	9·4771	9·4771	9·4771
$+\log \left(\frac{1}{l_1}\right)^n$			8·6198 *n*		8·6198 *n*	7·2396
log sum	0·0957	0·8620 *n*	9·6204 *n*	1·1439	0·2205	8·6537
Antilogs	1·247	$-7·278\ R_1$	$-0·417$	$+13·928 R_1^2$	$+1·662 R_1$	$+0·045$

Collection of terms	1·247	$-7·278\ R_1$
	$+0·045$	
	1·292	$+1·662\ R_1$
	$-0·417$	

$$LA'p = y^2 \left\{ 0·875 \quad -5·616\ R_1 \quad +13·928\ R_1^2 \right\}$$

In the above equation, by putting R_1 equal to various curvatures in turn, and y equal to the original semi-aperture of 0·8375, we shall obtain the primary spherical aberration given by each of the selected lens-shapes as before :—

"Bending" or lens-shape	a	b	c	d	e	f	g
R_1 =	$-0·10$	0·00	$+0·10$	$+0·15$	$+0·20$	$+0·30$	$+0·40$
Sph. Ab. =	$+1·105$	$+0·614$	$+0·317$	$+0·243$	$+0·217$	$+0·311$	$+0·601$

These values, it will be seen, agree fairly closely with those given by the ray-tracing method, but they are obtained with an enormous saving in time. It must be remembered, however, that with complex lens systems the analytical methods will not give particularly accurate results ; nevertheless they can be a distinctly helpful guide to the solution of a problem.

Calculation of Coma

It is not always sufficient to eliminate or reduce to a minimum the spherical aberration given by a lens, for it is often necessary to take into consideration simultaneously the amount of the aberration known as coma. Coma is usually spoken of as an oblique aberration and is caused by rays from the outer portions of a lens not coinciding with the principle oblique ray (see Fig. 9) ; this produces

a flared, comet-like patch of light instead of a uniformly illuminated circular patch. The length of the small vertical line in the figure is a measure of the amount of coma given by the lens. Obviously this will be a function of the height H'_K (or in other words, the obliquity angle U'_{pr}) and the aperture of the lens.

As we have not learnt as yet to trace oblique rays, the coma will be determined by means of the following analytical formula (see Conrady's *Applied Optics*, page 324) :—

The Coma Contribution $CC' = H'_K . \text{SA}^2 . \left\{\frac{1}{4}G_5 \mathcal{R} R_1 - G_7 \mathcal{R}\frac{1}{l_1} - G_8 \mathcal{R}^2\right\}$

where

H'_K is the image height (in this case assumed as 1 inch)

SA the semi-aperture of the lens $= y = 0 \cdot 8375$

$\mathcal{R}$ the total curvature of the lens $= 0 \cdot 300$

l_1 the object distance $= -24 \cdot 000$ inches

and G the functions already mentioned on page 19.

A simple calculation (8) given below will enable the numerical value of the coma to be obtained for various "bendings" of the lens :

CALCULATION No. 8

Formula: $H'_K . SA^2$	$\left\{\frac{1}{4}G_5\mathcal{R}R_1\right.$	$- G_7\mathcal{R}\frac{1}{l_1}$	$\left.- G_8\mathcal{R}^2\right\}$
$\log \frac{1}{4}$	9·3979		
+ $\log H'_K . \text{SA}^2$	9·8460	9·8460	9·8460
+ $\log G$	0·2351	9·8380 *n*	9·5945 *n*
+ $\log \mathcal{R}^n$	9·4771	9·4771	8·9542
+ $\log \frac{1}{l_1}$		8·6198 *n*	
log sum	8·9561	7·7809	8·3947 *n*
antilogs	0·0904 R_1	+0·0060	−0·0248

Thus the total $CC' = 0 \cdot 0904R_1 - 0 \cdot 0188$ and by substituting for R_1 the various curvature values as before we shall obtain the coma (sagittal) for each of the lens-shapes :—

Bendings	a	b	c	d	e	f	g
R_1 =	−0·10	0	+0·10	+0·15	+0·20	+0·30	+0·40
Coma =	−0·028	−0·019	−0·010	−0·005	−0·001	+0·008	+0·017

If these values are plotted, as in the lower half of Fig. 8, it will be seen that the coma plots as a straight line and further that the minimum numerical amount of coma occurs when R_1 is most suitable for the least spherical aberration.

Checks

It is obviously important to have some means of verifying the numerical work in the computing columns, especially if the ray-tracing is to be done through a number of surfaces; and therefore it is advisable to have checks whenever possible in order to satisfy oneself as to the reliability of the work already carried out.

3 — 6 — 9 *check*

A very simple check on the arithmetical work for a limited stage of the calculation is to add together two of the log values given in the third, sixth and

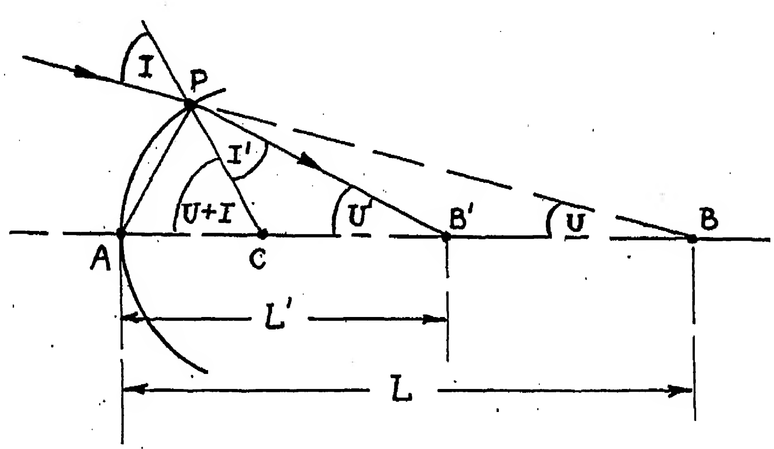


Fig. 10.

ninth line of the computing columns, namely those opposite $\log (L - r) \sin U$, $\log \frac{N}{N'}$, and $\log r \,.\, \sin I'$, and this sum should give the third log value. For example, in Calculation No. 1 (first surface) we find the logs $0{\cdot}74648$, $9{\cdot}81873$ and $0{\cdot}56521$, the sum of the first two giving the third value.

P.A. check

This check is the means of obtaining the intersection length L' by an entirely different route from the usual computing formulæ, and consists in locating the point P (Fig. 10) on the surface concerned, which fact enables the intersection length L' to be obtained quite independently of the radius r. Moreover the method employed has the effect of increasing the accuracy in locating the point B′ and is particularly useful when r happens to be a long radius; therefore the numerical value for L' as determined by the *PA*-check method is the one carried up into the next column as the new L. The derivation

of the check formulæ is given on pages 23–25 of Conrady's *Applied Optics* and those to be employed here are as follows :—

$$\left.\begin{aligned} PA &= L \,.\, \sin U \,.\, \sec \frac{I-U}{2} \\ PA &= L' \,.\, \sin U' \,.\, \sec \frac{I'-U'}{2} \end{aligned}\right\} \qquad (1)$$

$$L' = PA \,.\, \operatorname{cosec} U' \,.\, \cos \frac{I'-U'}{2} \qquad (2)$$

For paraxial rays $l' = \dfrac{l \,.\, u}{u'}$ . . . . (3)

Check Formulæ for Plane Surfaces

For marginal rays : $L' = L - L \sec U \,.\, \dfrac{\sin U' - U}{\sin U'}$

For paraxial rays : $l' = l - l \left(\dfrac{N - N'}{N}\right)$

At this stage it is a good thing to collect all the ray-tracing formulæ and the checks, and to mark out the various symbols and terms on separate white cardboard templets (see Fig. 11) so that they may be laid against the side of the computing columns instead of having to write down the symbols each time.

Achromatism

Hitherto we have dealt with single uncorrected lenses in which there are inherently present the three aberrations, chromatic, spherical and coma. An attempt will now be made to correct these, and the one which must be attended to first is the chromatic aberration. In order to illustrate the way in which this may be done, let us take an example in which we require a two-lens achromatic visual objective (for use with parallel light) having a focal length of ten inches, and consisting of crown and flint glasses with the following optical constants :—

	N_D	$N_F - N_C$	V	$N_D - N_C$	$N_F - N_D$
Glass for crown lens	1·5407	0·00910	59·4	0·00268	0·00642
Glass for flint lens	1·6225	0·01729	36·0	0·00492	0·01237

The first part of the procedure is to determine the total curvature of each component which, when combined, will give the correct focal length and complete achromatism. This can be done by putting the various numerical

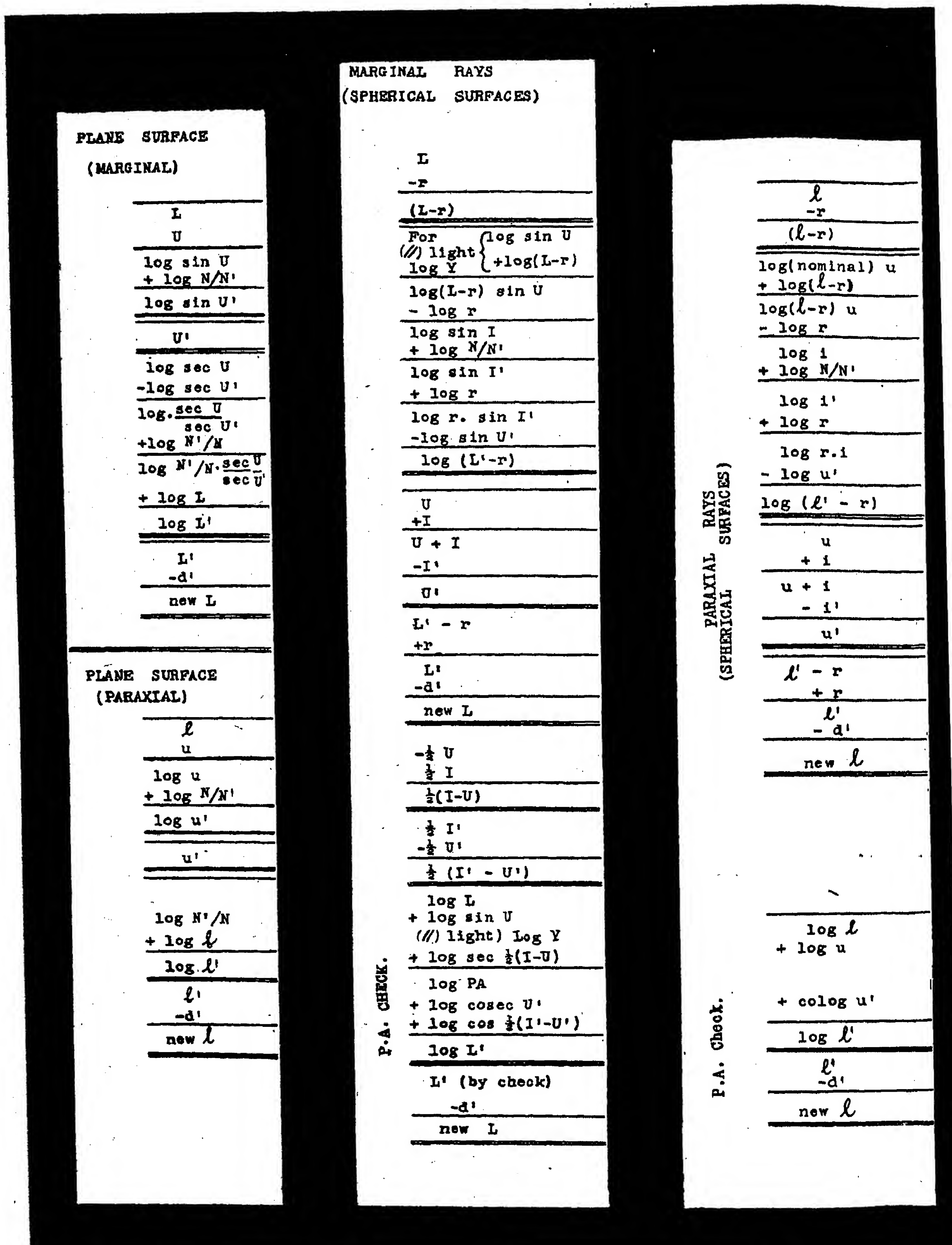


Fig. 11. Templets for use with the computing columns.

quantities into the analytical equation T.L. Chr. 4* (see Conrady's *Applied Optics*, page 148) as follows :—

$$\text{TL. Chr. 4*} \quad \begin{cases} R_a = \dfrac{1}{f'} \cdot \dfrac{1}{V_a - V_b} \cdot \dfrac{1}{\delta N_a} \\[2ex] R_b = \dfrac{1}{f'} \cdot \dfrac{1}{V_b - V_a} \cdot \dfrac{1}{\delta N_b} \end{cases}$$

where R_a and R_b represent the total curvatures of the crown lens (a) and the flint lens (b) respectively

f' the focal length of the complete lens system,

V_a and V_b the reciprocal of the dispersive powers for each glass as given in the glass list,

and δN_a and δN_b the respective mean dispersions $(N_F - N_C)$ for each glass. Then

$$R_a = \frac{1}{10} \cdot \frac{1}{23 \cdot 4} \cdot \frac{1}{0 \cdot 00910} = +0 \cdot 4697$$

$$R_b = \frac{1}{10} \cdot \frac{1}{-23 \cdot 4} \cdot \frac{1}{0 \cdot 01729} = -0 \cdot 2472.$$

By definition $R_a = \dfrac{1}{r_1} - \dfrac{1}{r_2}$ and $R_b = \dfrac{1}{r_3} - \dfrac{1}{r_4}$, therefore we can now select any number of crown and flint lenses which satisfy the condition of achromatism and give the desired focal length of the complete lens system.

A very common choice is to make the crown lens equi-convex, in which case

$$\frac{1}{r_1} = -\frac{1}{r_2} = \frac{1}{2} R_a = 0 \cdot 2348 \quad \text{and therefore}$$

$$r_1 = +4 \cdot 260 \text{ inches and } r_2 = -4 \cdot 260 \text{ inches.}$$

In objectives of this type it is frequently desirable that the crown and flint lenses be cemented together, and then r_3 will have the same radius as r_2, so that

$$R_b = \frac{1}{r_3} - \frac{1}{r_4} \qquad \text{now becomes}$$

$$-0 \cdot 2472 = -0 \cdot 2348 - \frac{1}{r_4}$$

$$\therefore \quad r_4 = +80 \cdot 65 \text{ inches.}$$

Another common choice is that the last surface shall be plane, in which case

$$r_4 = \infty \quad \text{and } R_b = \frac{1}{r_3}$$

$$\therefore \quad 1/r_3 = -0 \cdot 2472 \quad \text{or} \quad r_3 = -4 \cdot 045 \text{ inches.}$$

And as r_2 is equal to r_3 for cemented objectives,

$$R_a = \frac{1}{r_1} - \frac{1}{r_2}$$

$$\frac{1}{r_1} = R_a + \frac{1}{r_2} = 0 \cdot 4697 - 0 \cdot 2472 = +0 \cdot 2225$$

$$\therefore \quad r_1 = 4 \cdot 495 \text{ inches.}$$

It is interesting to note from these two particular examples that with an equi-convex crown component the last radius is very long (namely, about 80 inches) and one might be tempted to substitute a flat surface for r_4. This however, is not justified in practice for trigonometrical ray-tracing tests show that the residual aberrations exceed the permissible tolerances when this is done. Moreover, it will be noticed that for a cemented objective when r_4 is made equal to infinity, the crown lens is certainly not equi-convex and has in fact radii which differ by quite a considerable amount, namely, half an inch.

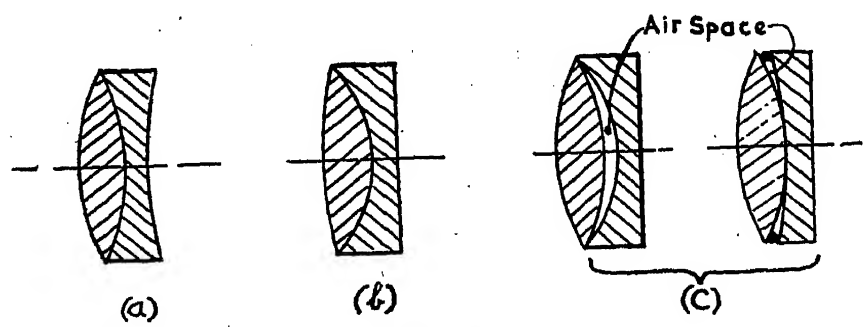


Fig. 12.

If, however, we wish to maintain an equi-convex crown lens and the flint lens with a flat last surface—the case which makes the task of the optical workshop easier—then there must be a thin air space (or air lens) between the contact surfaces and no cementing therefore is desirable. The above three types of objective are illustrated diagrammatically in Fig. 12 (a), (b) and (c). There are, of course, other shapes of the lenses which when combined will give the required chromatic correction, but these are dealt with in the section on the systematic design of telescope objectives.

Prescribed Amounts of Chromatic Aberration

In the foregoing examples it has been assumed that *complete* achromatism of the objective is required; this may not always be the case; for instance, if the complete instrument is to be used with an ordinary Huygenian or Ramsden eyepiece, the undercorrected chromatic aberration present in such eyepieces must be compensated for in the design of the object glass by introducing an amount of chromatic aberration of equal and opposite* sign to that given by the eyepiece.

* i.e., if the ray-tracing calculations for the eyepiece have been carried out in a *left-to-right* direction.

As an example, let us assume that an objective of the same focal length (namely, ten inches) and of the same glasses is called for, but that a chromatic over-correction $l'_r - l'_v = -0\cdot066$ inches is required at its focus. This value is one which has been obtained from calculations made on the Huygenian eyepiece given in the section on eyepiece design, and therefore will serve conveniently here.

In this case we make use of the formula T.L. Chr. (4), viz. :—

$$R_a = \frac{1}{f'(V_a - V_b)\delta N_a} - \frac{\text{Res. Chr. Ab. Term}}{\delta N_a} \cdot \frac{V_b}{V_a - V_b}$$

$$R_b = \frac{1}{f'(V_b - V_a)\delta N_b} - \frac{\text{Res. Chr. Ab. Term}}{\delta N_b} \cdot \frac{V_a}{V_b - V_a}$$

where the Residual Chromatic Aberration Term $= \left(\frac{l'_r - l'_v}{l'_r \,.\, l'_v}\right)_b - \left(\frac{l_r - l_v}{l_r \,.\, l_v}\right)_a$

We begin by calculating the Residual Chromatic Aberration term, but as no chromatic aberration at the distant object is specified the formula reduces to the first bracket in which the numerator $l'_r - l'_v = -0\cdot066$; and as the objective is used at its principal focus both l'_r and l'_v will be very nearly identical with f' and therefore we may write :—

$$\text{Res. Chrom. Ab. Term} = \frac{-0\cdot066}{(f')^2} = \frac{-0\cdot066}{100} = -0\cdot00066$$

so that the total curvatures R_a and R_b of the two components can now be calculated, giving

$$R_a = \left(\frac{1}{10 \times 23\cdot4 \times 0\cdot00910}\right) - \left(\frac{-0\cdot00066}{0\cdot00910} \times \frac{36\cdot0}{23\cdot4}\right) = +0\cdot5813$$

$$R_b = \left(\frac{1}{10 \times -23\cdot4 \times 0\cdot01729}\right) - \left(\frac{-0\cdot00066}{0\cdot01729} \times \frac{59\cdot4}{-23\cdot4}\right) = -0\cdot3441$$

It will be noticed that the slight over-correction required increases the total curvature quite considerably, when compared with those obtained for complete achromatism, namely $+0\cdot4697$ and $-0\cdot2472$. And if we calculate the data for a cemented objective having a plane last surface, we find :—

$$r_4 = \infty$$

$$R_b = \frac{1}{r_3} - \frac{1}{r_4}; \quad -0\cdot3441 = \frac{1}{r_3} - \frac{1}{\infty}; \text{ and therefore } r_3 = -2\cdot906 \text{ inches}$$

$$r_2 = -2\cdot906 \text{ inches}$$

$$R_a = \frac{1}{r_1} - \frac{1}{r_2}; \quad +0\cdot5813 = \frac{1}{r_1} - (-0\cdot3441); \quad \therefore\ r_1 = +4\cdot216 \text{ inches}$$

Hence, the radii of the surfaces become shorter than those in the previous example, more especially the contact surface which is now $-2\cdot906$ inches as compared with $-4\cdot045$ inches.

It should be remembered, however, that if a fully corrected eyepiece is going to be used with the objective, the latter will be designed for complete achromatism and the remarks contained in this section will not apply. But as many instruments are still fitted with uncorrected forms of eyepiece, it is important to know the procedure in such cases.

Trigonometrical Ray-tracing through an Achromatic Lens

Having covered some of the preliminary ground-work, it will be found advisable to carry out a full-scale ray-tracing test (including checks) on an achromatic lens system in order to find the exact amount of the aberrations, and we cannot do better than to take our telescope objective as calculated by thin lens formulæ on page 25 and see what kind of result this gives. The specification will be as follows :—

Focal length $f' = 10$ inches. $r_1 = +4\cdot260$ inches.

Clear aperture $= 1\cdot60$ inches. $r_2 = r_3 = -4\cdot260$ inches.

$r_4 = +80\cdot650$ inches.

Axial thicknesses $\begin{cases} \text{crown lens} = 0\cdot25 \text{ inches} \\ \text{flint lens} = 0\cdot20 \text{ inches} \end{cases}$

In order to test the chromatic aberration, C (red) and F (blue) rays will be traced at $\sqrt{0\cdot5}$ of the semi-aperture, and to test the spherical aberration an extreme marginal, a zonal, and a paraxial ray will be traced in " brightest light ", namely that part of the spectrum ($\lambda = 5555$A.) to which the eye is most sensitive.

We shall require to know, therefore, the refractive indices of the two glasses for these various wavelengths, and by reference to the glass list and a simple calculation, we find :—

	N_D	N_C	N_F	N_{yg}
Crown glass	1·5407	1·5380	1·5471	1·5424
Flint glass	1·6225	1·6176	1·6349	1·6257 (5)

N.B. The refractive index N_{yg} for the yellow-green part of the spectrum may be obtained from $N_{yg} = N_D + 0\cdot188\,(N_F - N_C)$.

Testing the chromatic aberration first, we start with an initial white ray parallel to the axis at a height $Y = \sqrt{0\cdot5} \times 0\cdot80 = 0\cdot5657$ and the ray-tracing now becomes as in Calculation No. 9, which includes the use of the PA-check.

Calculation No. 9

	1st SURFACE C ray	1st SURFACE F ray	2nd SURFACE C ray	2nd SURFACE F ray	3rd SURFACE C ray	3rd SURFACE F ray
L	Parallel Rays		11·8817	11·7514	14·4666	14·5914
$-r$			+4·260	4·260	−80·650	−80·650
$(L-r)$			16·1417	16·0114	−66·1834	−66·0586
log sin U	$Y = 0{\cdot}5657$		8·66954	8·67424	8·57851	8·57477
+ log $(L-r)$			1·20795	1·20443	1·82075 *n*	1·81993 *n*
log $(L-r)$ sin U	9·75259	same	9·87749	9·87867	0·39926 *n*	0·39470 *n*
− log r	0·62941		0·62941 *n*	0·62941 *n*	1·90660	1·90660
log sin I	9·12318	9·12318	9·24808 *n*	9·24926 *n*	8·49266 *n*	8·48810 *n*
+ log $\left(\frac{N}{N'}\right)$	9·81304	9·81048	9·97809	9·97603	0·20887	0·21349
log sin I'	8·93622	8·93366	9·22617 *n*	9·22529 *n*	8·70153 *n*	8·70159 *n*
+ log r	0·62941	0·62941	0·62941 *n*	0·62941 *n*	1·90660	1·90660
log r . sin I'	9·56563	9·56307	9·85558	9·85470	0·60813 *n*	0·60819 *n*
− log sin U'	8·66954	8·67424	8·57851	8·57477	8·75656	8·75660
log $(L'-r)$	0·89609	0·88883	1·27707	1·27993	1·85157 *n*	1·85159 *n*
U	0– 0– 0	0– 0– 0	2–40–41	2–42–26	2–10–17	2– 9–10
$+ I$	7–37–52	7–37–52	−10–11–51	−10–13–32	− 1–46–54	− 1–45–47
$U + I$	7–37–52	7–37–52	− 7–31–10	− 7–31– 6	0–23–23	0–23–23
$-I'$	4–57–11	4–55–26	+9–41–27	+9–40–16	+2–52–59	+2–53– 0
U'	2–40–41	2–42–26	2–10–17	2– 9–10	3–16–22	3–16–23
$(L'-r)$	7·8721	7·7416	18·9265	19·0515	− 71·0510	− 71·0542
$+ r$	4·260	4·260	− 4·260	− 4·260	80·650	80·650
L'	12·1321	12·0016	14·6665	14·7915	9·5990	9·5958
$-d'$	0·250	0·250	0·200	0·200		
new L	11·8821	11·7516	14·4665	14·5915		
$-\frac{1}{2} U$	0– 0– 0	0– 0– 0	− 1–20–20	− 1–21–13	− 1– 5– 8	− 1– 4–35
$+\frac{1}{2} I$	3–48–56	3–48–56	− 5– 5–56	− 5– 6–46	− 0–53–27	− 0–52–54
$\frac{1}{2}(I-U)$	3–48–56	3–48–56	− 6–26–16	− 6–27–59	− 1–58–35	− 1–57–29
$\frac{1}{2} I'$	2–28–36	2–27–43	− 4–50–44	− 4–50– 8	− 1–26–30	− 1–26–30
$-\frac{1}{2} U'$	1–20–20	1–21–13	− 1– 5– 8	− 1– 4–35	− 1–38–11	− 1–38–12
$\frac{1}{2}(I'-U')$	1– 8–16	1– 6–30	− 5–55–52	− 5–54–43	− 3– 4–41	− 3– 4–42
log L			1·07488	1·07009	1·16037	1·16410
+ log sin U			8·66954	8·67424	8·57851	8·57477
log Y (for // light)	9·75259	9·75259				
+ log sec $\frac{1}{2}(I-U)$	0·00096	0·00096	0·00275	0·00277	0·00026	0·00025
log PA	9·75355	9·75355	9·74717	9·74710	9·73914	9·73912
+ log cosec U'	1·33046	1·32576	1·42149	1·42523	1·24344	1·24340
+ log cos $\frac{1}{2}(I'-U')$	9·99991	9·99992	9·99767	9·99768	9·99937	9·99937
log L'	1·08392	1·07923	1·16633	1·17001	0·98195	0·98189
L' (by check)	12·1317	12·0014	14·6666	14·7914	9·5929	9·5916
$-d'$	0·250	0·250	0·200	0·200		
new L	11·8817	11·7514	14·4666	14·5914		

The considerable disagreement in columns 5 and 6 between the values given by the ordinary calculation and those given by the *PA*-check is due to the fact that r_3 is a very long radius. The intersection lengths given by the *PA*-check method (as explained earlier) are always more accurate, but especially so when a long radius is used ; and therefore the values $L'_C = 9 \cdot 5929$ and $L'_F = 9 \cdot 5916$ are the ones to be relied on.

Thus, the Chromatic Aberration $(L'_C - L'_F) = + 0 \cdot 0013$ (undercorrected).

The spherical aberration, both marginal and zonal, will now be tested ; this involves tracing incident marginal, zonal and paraxial rays in one colour, namely N_{yg}, equal to $1 \cdot 5424$ and $1 \cdot 62575$ for the crown and flint glasses respectively. This is shown in Calculation No. 10.

From this ray-tracing the marginal spherical aberration

$$(l' - L'_M) = 9 \cdot 5958 - 9 \cdot 5830 = + 0 \cdot 0128 \text{ inches}$$

and the zonal spherical aberration $(l' - L'_Z) = 9 \cdot 5958 - 9 \cdot 5880 = + 0 \cdot 0078$ inches.

It is useful to know the equivalent focal length of the lens system, and this may be obtained from log y − log final $u' = 9 \cdot 90309 - 8 \cdot 90601 = 0 \cdot 99708$ and the antilog of this gives $f' = 9 \cdot 933$ inches.

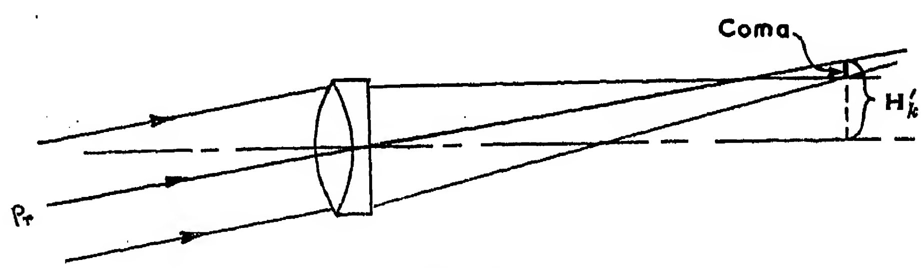


Fig. 13.

In carrying out these tests on our objective, there is one other thing we wish to know, namely the extent of the coma. As mentioned previously, in order to obtain the exact amount of coma present, it would be necessary to trace three oblique rays through the lens at a given H'_k (see Fig. 13), but although we have not yet done oblique ray-tracing we can obtain a very close approximation to the true value, by applying an analytical formula for the offence against the sine condition discussed at length on pages 367 to 370 of Conrady's *Applied Optics*. The offence against the sine condition can be expressed as the ratio of the coma to the image height H'_k and can be shown to be equal to :—

$$OSC' = 1 - \left(\frac{\sin U_1}{u_1}\right) \cdot \frac{u'_k}{\sin U'_k} \cdot \frac{l'_k - l'pr_k}{L'_k - l'pr_k}$$

where $l'pr_k$ is the distance of the stop or diaphragm from the lens, and the suffixes k refer to the intersection lengths and angles after the rays have passed through the *last* surface.

To face page 30.

CALCULATION No. 10

	1st SURFACE			*2nd SURFACE*			*3rd SURFAC*		
	Marginal	*Paraxial*	*Zonal*	*Marginal*	*Paraxial*	*Zonal*	*Marginal*	*Paraxial*	
L $-r$		Parallel Rays		11·7732 4·260	11·8638 4·260	11·8181 4·260	14·5214 –80·650	14·5191 –80·650	
$(L-r)$				16·0332	16·1238	16·0781	–66·1286	–66·1309	
log sin U + log $(L-r)$	$Y = 0·8$	Nominal $y = 0·8$	$Y = 0·5657$	8·82485 1·20502	8·81981 1·20747	8·67183 1·20624	8·72869 1·82039 *n*	8·72615 1·82040 *n*	
log $(L-r)$ sin U – log r	9·90309 0·62941		9·75259 0·62941	0·02987 0·62941 *n*	0·02728 0·62941 *n*	9·87807 0·62941 *n*	0·54908 *n* 1·90660	0·54655 *n* 1·90660	
log sin I + log $\left(\frac{N}{N'}\right)$	9·27368 9·81180	same	9·12318 9·81180	9·40046 *n* 9·97715	9·39787 *n* 9·97715	9·24866 *n* 9·97715	8·64248 *n* 0·21105	8·63995 *n* 0·21105	
log sin I' + log r	9·08548 0·62941		8·93498 0·62941	9·37761 *n* 0·62941 *n*	9·37502 *n* 0·62941 *n*	9·22581 *n* 0·62941 *n*	8·85353 *n* 1·90660	8·85100 *n* 1·90660	
log r . sin I' – log sin U'	9·71489 8·82485	9·71489 8·81981	9·56439 8·67183	0·00702 8·72869	0·00443 8·72615	9·85522 8·57695	0·76013 *n* 8·90846	0·75760 *n* 8·90601	
log $(L'-r)$	0·89004	0·89508	0·89256	1·27833	1·27828	1·27827	1·85167 *n*	1·85159 *n*	
U + I	0– 0– 0 10–49–27	0·000000 0·187793	0– 0– 0 7–37–52	3–49–51 – 14–33–49	0·066040 – 0·249959	2–41–32 – 10–12–41	3– 4– 9 – 2–30–58	0·053229 – 0·043647	
$U + I$ $- I'$	10–49–27 6–59–36	0·187793 0·121753	7–37–52 4–56–20	– 10–43–58 + 13–48– 7	– 0·183919 + 0·237148	– 7–31– 9 + 9–40–58	0–33–11 4– 5–34	0·009582 0·070958	
U'	3–49–51	0·066040	2–41–32	3– 4– 9	0·053229	2– 9–49	4–38–45	0·080540	
$(L'-r)$ + r	7·7632 4·260	7·8538 4·260	7·8084 4·260	18·9815 – 4·260	18·9793 – 4·260	18·9789 – 4·260	– 71·0673 + 80·650	– 71·0542 80·650	–
L' $- d'$	12·0232 0·250	12·1138 0·250	12·0684 0·250	14·7215 0·200	14·7193 0·200	14·7189 0·200	9·5827	9·5958	
new L	11·7732	11·8638	11·8184	14·5215	14·5193	14·5189			
$-\frac{1}{2} U$ $\frac{1}{2} I$	0– 0– 0 5–24–44		0– 0– 0 3–48–56	– 1–54–56 – 7–16–54		– 1–20–46 – 5– 6–20	– 1–32– 4 – 1–15–29		– –
$\frac{1}{2}(I-U)$	5–24–44		3–48–56	– 9–11–50		– 6–27– 6	– 2–47–33		–
$\frac{1}{2} I'$ $-\frac{1}{2} U'$	3–29–48 1–54–56		2–28–10 1–20–46	– 6–54– 4 – 1–32– 4		– 4–50–29 – 1– 4–54	– 2– 2–47 – 2–19–22		– –
$\frac{1}{2}(I'-U')$	1–34–52		1– 7–24	– 8–26– 8		– 5–55–23	– 4–22– 9		–
log L + log sin U log Y (for // light) + log sec $\frac{1}{2}(I-U)$	9·90309 0·00194	9·90309	9·75259 0·00096	1·07089 8·82485 0·00562	1·07422 8·81981	1·07255 8·67183 0·00276	1·16201 8·72869 0·00052	1·16194 8·72615	1· 8· 0·
log PA + log cosec U' + log cos $\frac{1}{2}(I'-U')$	9·90503 1·17515 9·99984	1·18019	9·75355 1·32817 9·99992	9·90136 1·27131 9·99528	9·89403 1·27385	9·74714 1·42305 9·99768	9·89122 1·09154 9·99874	9·88809 1·09399	9· 1· 9·
log L'	1·08002	1·08328	1·08164	1·16795	1·16788	1·16787	0·98150	0·98208	0·
L' (by check) $- d'$	12·0232 0·250	12·1138 0·250	12·0681 0·250	14·7214 0·200	14·7191 0·200	14·7187 0·200	9·5830	9·5958	9·
new L	11·7732	11·8638	11·8181	14·5214	14·5191	14·5187			

For a telescope objective, the diaphragm may be considered as being *on* the surface of the lens and therefore $l'pr_k$ will be zero in this case; moreover for incident parallel light the quantity $\left(\dfrac{\sin U_1}{u_1}\right)$ becomes $\left(\dfrac{Y}{y}\right)$ and as we choose in usual practice $y = Y$ this term may be omitted, so that our equation now becomes :—

$$OSC' = 1 - \frac{u'_k \, . \, l'_k}{\sin U'_k \, . \, L'_k}$$

Applying the numerical values from our ray-tracing, we get :—

$$OSC' = 1 - \frac{0 \cdot 080540 \times 9 \cdot 5958}{\sin 4^\circ\text{–}38'\text{–}45'' \times 9 \cdot 5830}$$

$$\begin{array}{lr} \log 0 \cdot 080540 & = 8 \cdot 90601 \\ + \log 9 \cdot 5958 & = 0 \cdot 98208 \\ + \text{cosec } 4\text{–}38'\text{–}45'' & = 1 \cdot 09154 \\ + \text{colog } 9 \cdot 5830 & = 9 \cdot 01850 \\ \hline \log \text{ 2nd term} & = 9 \cdot 99813 \\ \hline \therefore \text{ 2nd term} & = 0 \cdot 99570 \end{array}$$

$$OSC' = 1 - 0 \cdot 99570$$

$$OSC' = + 0 \cdot 00430$$

Optical Tolerances

In all our ray-tracing methods we obtain numerical values for the lack of union of rays either longitudinally along the axis or at right angles to the axis of the optical system concerned, and it is essential to know to what extent these errors affect the definition of the image, this latter of course being the final criterion of the optical performance of any instrument.

In other words, we must know the tolerances which can be permitted for these aberrations so that the definition may not be considered defective. This physical aspect of the optical image has been studied by a number of workers, but the original work on this subject by the late Lord Rayleigh in 1878 still appears valid, namely that if the difference between the longest and shortest ray paths leading to a selected focus does not exceed one quarter of the wavelength of light, then the optical instrument would not fall seriously short of giving a perfect performance.

When the Rayleigh quarter-wave limit is accepted, its utility depends on finding simple formulæ which enable the optical designer to convert the residual aberrations determined geometrically into a form which can be

recognized as quality of definition. These have been deduced by Conrady (see page 136 of his *Applied Optics*) and are as follows :—

(1) Focal Range $= \dfrac{1 \text{ wavelength}}{N' \,.\, \sin^2 U'_M}$

where U'_M is the angle the extreme marginal ray makes with the axis, and N' the refractive index in the image space.

[N.B. The wavelength of light λ may be taken as $0 \cdot 000022$ inch or $0 \cdot 00055$ mm.]

(2) The Chromatic Aberration Tolerance is the same as that for the focal range when U'_M is used, but is equal to $\pm \dfrac{0 \cdot 5\lambda}{N' \,.\, \sin^2 U'_{0 \cdot 7071}}$ when the ($\sqrt{0 \cdot 5}$ of the aperture) zone is employed.

(3) Spherical Aberration Tolerance $= \pm \dfrac{4\lambda}{N' \,.\, \sin^2 U'_M}$

(4) Zonal Spherical Aberration Tolerance (for the $\sqrt{0 \cdot 5}$ of the aperture zone) $= \pm \dfrac{6\lambda}{N' \,.\, \sin^2 U'_M}$

in the case when the extreme marginal and paraxial rays come to a common focus.

(5) Coma Tolerance $= \pm \dfrac{\frac{1}{2}\lambda}{N' \,.\, \sin U'_M}$

and the Offence against the Sine Condition (OSC')

$$= \pm \frac{\frac{1}{2}\lambda}{N' \,.\, H'_K \,.\, \sin U'_M}$$

It should be noted that these two last named tolerances have simply U'_M in the denominator, whilst all the previous ones had the square of U'_M; this is due to the fact that we measure coma as a transverse aberration whilst the others are longitudinal ones. The Rayleigh limit is very severe in the case of coma, but when the actual linear size of the coma patch has to be considered, then :—

The admissible coma may be $\begin{cases} 0 \cdot 001 \text{ inch for extremely sharp definition.} \\ 0.004 \text{ inch for good definition.} \\ 0 \cdot 010 \text{ inch for ``soft'' definition.} \end{cases}$

For ordinary telescopes, and microscopes, where the angular field is not large, we may take the admissible OSC' as $\pm 0 \cdot 0025$.

Every endeavour should be made to keep the aberrations obtained trigonometrically to within the above tolerances, but when more aberrations are included in our problem it becomes increasingly difficult to satisfy

every tolerance simultaneously, for example, in the design of photographic lenses the spherical aberration tolerance may have to be exceeded in order to correct some other aberration. Thus experience must be used in order to strike a balance for the most favourable compromise.

If we now apply these tolerances to the ray-tracing calculations made on the objective we have been testing we shall be able to see whether the design will give good definition or not.

Taking, first of all, the chromatic aberration the tolerance will be $\frac{1\lambda}{N' . \sin^2 U'_M}$, where $\lambda = 0 \cdot 000022$ inch and U'_M (from Calculation No. 10 column 7) $= 4°–38'–45''$, hence the chromatic aberration tolerance $= \pm 0 \cdot 0034$ inch. The actual chromatic aberration obtained from Calculation No. 9 is $+0 \cdot 0013$ inch, so that the lens is well corrected as far as colour is concerned.

The marginal spherical aberration tolerance will be

$\frac{4 \times 0 \cdot 000022}{1 \times \sin^2 (4°–38'–45'')} = \pm 0 \cdot 0136$ inch, whereas from Calculation No. 10 the actual value is $+0 \cdot 0128$ inch.

The coma tolerance (from the Rayleigh limit) $\pm \frac{\frac{1}{2}\lambda}{N' . \sin U'_M}$ will be $\pm 0 \cdot 00014$ inch, and as we have taken H'_K in our problem to be one inch, the *OSC′* tolerance would be the same as that for the coma. But we may quite safely adopt the previously stated *OSC′* tolerance value of $\pm 0 \cdot 0025$ for practically all telescope objectives and the value for the *OSC′* as calculated on page 31 gives $+ 0 \cdot 0043$.

Analysing the results, we see that whilst our objective is quite well corrected for colour and marginal spherical aberration, the coma is greater than the permissible tolerance.

The aberration of $+0 \cdot 0043$ for the *OSC′* is certainly too high and should not be allowed to pass.

The way in which these defects may be adjusted will be seen in the following section on "the systematic design of telescope objectives".

CHAPTER II

THE SYSTEMATIC DESIGN OF TELESCOPE OBJECTIVES

THIS section deals with the systematic design of telescope objectives, as distinct from the somewhat haphazard choice of a possible solution made in the previous chapter. The procedure involves some of the principles already dealt with, but also includes a number of new ones which together tend to produce a "best possible" solution to the problem.

It is as well, therefore, at the outset to set down the sequence of operations we shall use :—

(1) Choose a number of suitable pairs of glasses from the glass catalogue.

(2) For each pair of glasses determine the curvature or power of the components to give the prescribed focal length and chromatic condition.

(3) Employ the analytical method for the determination of the total spherical aberration given by the lens system.

(4) Plot the spherical aberration against the curvature of the contact surface, after substituting suitable values for R_2 in the equation given in (3).

(5) Determine the coma contribution (CC') equation by the analytical method, and by suitable substitution for R_2 obtain values for plotting as in (4).

(6) The operations contained in paragraphs 4 and 5 should, of course, be carried out for each pair of glasses chosen, and all the curves (both spherical aberration and coma) plotted on one piece of graph paper.

(7) From the graphs, determine which pair of glasses and what value for R_2 are most suitable for giving minimum spherical aberration and coma simultaneously, or as specified.

(8) Test this solution by trigonometrical ray-tracing, and apply the optical tolerances.

(9) Improve, if necessary, the chromatic aberration, by adjustment of the last radius.

(10) If it be necessary, improve the spherical aberration by "bending" the lens system as a whole.

(11) Test the "Offence against the Sine Condition".

For the purpose of illustrating this procedure, let us ask that a telescope objective of ten inches focal length and one inch aperture be designed, and that it shall be free from chromatic aberration, spherical aberration and coma.

Complying, therefore, with our schedule we will select (say) four likely pairs of glasses from the glass list. It will be re-called from earlier theory that in order to secure good achromatism, the ratio of the partial dispersions to the mean dispersion $\left(\text{i.e., } \frac{N_D - N_C}{N_F - N_C};\ \frac{N_F - N_D}{N_F - N_C};\ \frac{N_{G'} - N_F}{N_F - N_C}\right)$ for the crown glass should be as near as possible equal to these corresponding values for the flint glass. Another consideration must also be taken into account, namely that of attempting to obtain a small amount of coma simultaneously with very little spherical aberration. The four glasses chosen here (Table II) will serve to illustrate these points as the work develops.

TABLE II

Pair No.	*Glasses*	N_D	*Mean Dispersion* $(N_F - N_C)$	V	*Partial Dispersions* $(N_D - N_C)$	$(N_F - N_D)$	$(N_{G'} - N_F)$
(1)	Hard Crown	1·51750	0·00856	60·5	0·00250	0·00606	0·00484
	Dense Flint	1·62250	0·01729	36·0	0·00492	0·01237	0·01047
(2)	Medium Barium Crown	1·57211	0·00991	57·7	0·00290	0·00701	0·00567
	Dense Flint	1·61661	0·01686	36·6	0·00481	0·01205	0·01021
(3)	Hard Crown	1·51750	0·00856	60·5	0·00250	0·00606	0·00484
	Extra Dense Flint ..	1·69700	0·02287	30·5	0·00646	0·01641	0·01400
(4)	Hard Crown	1·51750	0·00856	60·5	0·00250	0·00606	0·00484
	Extra Dense Flint ..	1·65091	0·01940	33·6	0·00551	0·01389	0·01190

N.B.—In the table of refractive indices given above, N_D is used instead of the more modern N_d. This has been done in order to make these pages agree closely with Conrady's original text.

We will now proceed to determine the power of each component to give complete achromatism and a focal length of ten inches for the various pairs of glasses from :—

$$R_{crown} = \frac{1}{f'} \cdot \frac{1}{(V_a - V_b)} \cdot \frac{1}{\delta N_{crown}} \qquad R_{flint} = \frac{1}{f'} \cdot \frac{1}{(V_b - V_a)} \cdot \frac{1}{\delta N_{flint}}$$

For pair No. 1: $$R_a = \frac{1}{10 \times 24 \cdot 5 \times 0 \cdot 00856} = +0 \cdot 4768$$

$$R_b = \frac{1}{10 \times -24 \cdot 5 \times 0 \cdot 01729} = -0 \cdot 2361$$

For pair No. 2: $$R_a = \frac{1}{10 \times 21 \cdot 1 \times 0 \cdot 00991} = +0 \cdot 4782$$

$$R_b = \frac{1}{10 \times -21 \cdot 1 \times 0 \cdot 01686} = -0 \cdot 2811$$

For pair No. 3: $$R_a = \frac{1}{10 \times 30 \times 0 \cdot 00856} = +0 \cdot 3894$$

$$R_b = \frac{1}{10 \times -30 \times 0 \cdot 02287} = -0 \cdot 1457$$

For pair No. 4: $$R_a = \frac{1}{10 \times 26 \cdot 9 \times 0 \cdot 00856} = +0 \cdot 4342$$

$$R_b = \frac{1}{10 \times -26 \cdot 9 \times 0 \cdot 01940} = -0 \cdot 1916$$

The spherical aberration $LA'p$ contributed by each of the component lenses will now be calculated by the analytical method, which for two *thin* lenses may be obtained from equation T.L. (10), namely

$$LA'p = y^2 \cdot (l_b')^2 \left[G_1 \cdot R_a^3 + G_2 \cdot R_a^2 \cdot R_2 - G_3 \cdot R_a^2 \left(\frac{1}{l_2'}\right) + G_4 R_a R_2^2 - G_5 R_a R_2 \left(\frac{1}{l_2'}\right) + G_6 R_a \left(\frac{1}{l_2'}\right)^2 \right]$$
$$+ y^2 \cdot (l_b')^2 \left[G_1 R_b^3 - G_2 R_b^2 \cdot R_3 + G_3 R_b^2 \left(\frac{1}{l_3}\right) + G_4 R_b R_3^2 - G_5 R_b R_3 \left(\frac{1}{l_3}\right) + G_6 R_b \left(\frac{1}{l_3}\right)^2 \right]$$

where y is the semi-aperture of the lens for which the aberration is required.

(l_b') the image distance from lens b, in this case the focal length of the complete lens system, namely 10 inches.

R_a and R_b the total curvatures of the crown lens a and the flint lens b respectively.

R_2 and R_3 the curvatures of the contact surfaces, viz., $\frac{1}{r_2}$ and $\frac{1}{r_3}$

$\frac{1}{l_2'}$ is the reciprocal of the image distance after the light has passed through the first lens; and is obtained from $\frac{1}{l_2'} = (N_a - 1)R_a + \frac{1}{l_1}$ (N.B. l_1 is infinite; i.e., parallel light)

and the G-values are functions of the refractive indices as already explained, and obtained from Table I.

Applying this formula to the first pair of glasses and preparing the various numerical values for use with it, we have :—

$y^2.(l_b')^2 = 0{\cdot}5^2 \times 10^2 = 25$; $R_a = +0{\cdot}4768$; $N_a = 1{\cdot}5175$;

$\frac{1}{l_2'} = (N_a - 1)R_a + \frac{1}{l_1} = 0{\cdot}5175 \times 0{\cdot}4768$;

and $\therefore \log \frac{1}{l_2'} = 9{\cdot}3922.$

The Calculation (No. 11) is done in two parts, one for the crown lens and one for the flint :—

CALCULATION No. 11

Crown Lens	$G_1 . R_a^3$	$+G_2R_a^2R_2$	$-G_3R_a^2 . \frac{1}{l_2'}$	$+G_4R_a . R_2^2$	$-G_5R_a\frac{1}{l_2'}R_2$	$+G_6R_a . \left(\frac{1}{l_2'}\right)^2$
log G	9·7751	0·0188	0·1574n	9·7780	0·2347n	0·0482
+ log $y^2 . (l_b')^2$	1·3979	1·3979	1·3979	1·3979	1·3979	1·3979
+ log R_a^n	9·0349	9·3566	9·3566	9·6783	9·6783	9·6783
+ log $\left(\frac{1}{l_2'}\right)^n$			9·3922		9·3922	8·7844
log sum	0·2079	0·7733	0·3041n	0·8542	0·7031n	9·9088
antilogs	1·6140	5·9334R_2	−2·0142	7·1482R_2^2	−5·0478R_2	0·8106

Collection of Terms.

$$\begin{array}{rr} +0{\cdot}8106 & 5{\cdot}9334R_2 \\ +1{\cdot}6140 & -5{\cdot}0478R_2 \\ \hline +2{\cdot}4246 & \\ -2{\cdot}0142 & \end{array}$$

Sph. Ab. contributed by crown lens (a) $= +0{\cdot}410 + 0{\cdot}886\,R_2 + 7{\cdot}148\,R_2^2$ (to three significant figures)

Flint lens : $N_b = 1\cdot 6225$; $R_b = -0\cdot 2361$; $y^2(l_b')^2 = 25$; $\frac{1}{l_3} = \frac{1}{l_2'}$

and therefore $\log \frac{1}{l_3} = 9\cdot 3922$.

Flint Lens	$G_1 . R_b^3$	$-G_2 R_b^2 R_3$	$+G_3 R_b^2 . \frac{1}{l_3}$	$+G_4 R_b R_3^2$	$-G_5 R_b . \frac{1}{l_3} . R_3$	$+G_6 R_b \left(\frac{1}{l_3}\right)^2$
$\log G$	9·9135	0·1209n	0·2616	9·8420	0·3037n	0·1197
$+ \log y^2 . (l_b')^2$	1·3979	1·3979	1·3979	1·3979	1·3979	1·3979
$+ \log R_b^n$	8·1193n	8·7462	8·7462	9·3731n	9·3731n	9·3731n
$+ \log \left(\frac{1}{l_3}\right)^n$			9·3922		9·3922	8·7844
log sum	9·4307n	0·2650n	9·7979	0·6130n	0·4669	9·6751n
antilogs	$-0\cdot 2696$	$-1\cdot 8408 R_3$	0·6279	$-4\cdot 1020 R_3^2$	$2\cdot 9302 R_3$	$-0\cdot 4733$

Collection of Terms		
	$-0\cdot 4733$	$2\cdot 9302 R_3$
	$-0\cdot 2696$	$-1\cdot 8408 R_3$
	$-0\cdot 7429$	
	$+0\cdot 6279$	

Sph. Ab. contributed by flint lens (b) $\Big\} = -0\cdot 115 + 1\cdot 089\, R_3 - 4\cdot 102\, R_3^2$.

The result of these calculations is that the spherical aberration of our telescope objective can be determined for any independent "bendings" of the two components by putting the proper values of R_2 and R_3 into the equation :—

$$\text{Spherical Aberration } (LA'p) = 0\cdot 410 + 0\cdot 886\, R_2 + 7\cdot 148\, R_2^2 \text{ (crown)}$$
$$- 0\cdot 115 + 1\cdot 089\, R_3 - 4\cdot 102\, R_3^2 \text{ (flint)}.$$

For a *cemented* telescope objective where R_2 is equal to R_3, we can therefore simply add together the corresponding terms in the two lines of our equation ; and calling the contact curvature R_2, we obtain

$$\text{Sph. Ab. } (LA'p) = 0\cdot 295 + 1\cdot 975\, R_2 + 3\cdot 046\, R_2^2.$$

This is a quadratic equation in R_2 which can be solved (in the usual way) for any desired value of $LA'p$. It is instructive, however, to substitute suitable values for R_2 and find the corresponding amount of spherical aberration in each case ; and then to plot the resulting curve. For example,

with R_2 =	$-0\cdot 10$	$-0\cdot 15$	$-0\cdot 20$	$-0\cdot 25$	$-0\cdot 30$	$-0\cdot 35$	$-0\cdot 40$
Sph. Ab. =	0·128	0·068	0·022	$-0\cdot 008$	$-0\cdot 023$	$-0\cdot 023$	$-0\cdot 008$

These points are plotted in curve No. 1 of Fig. 14.

We will now proceed with the determination of the coma. This may be done by an analytical solution which gives the total coma contribution (CC') as :—

$$CC' = H'_k \cdot y^2 \left\{ \frac{1}{4} G^a_5 \cdot R_a R_2 - G^a_7 R_a \frac{1}{l'_2} + G^a_8 R^2_a + \frac{1}{4} G^b_5 R_b R_3 - G^b_7 R_b \frac{1}{l_3} - G^b_8 R^2_b \right\}$$

The only term we do not know in this equation is the image height H'_k, and for the sake of this example we will choose this as being equal to one inch.* The arrangement of the mathematical work is shown in Calculation No. 12.

Calculation No. 12

$H'_k \cdot y^2 = 1 \times 0\cdot5^2 = 0\cdot25$ $\left\{ \begin{matrix} N_a = 1\cdot5175 \\ N_b = 1\cdot6225 \end{matrix} \right\}$ $\left\{ \begin{matrix} R_a = +0\cdot4768 \\ R_b = -0\cdot2361 \end{matrix} \right\}$ $\log \frac{1}{l'_2}$ or $\frac{1}{l_3} = 9\cdot3922$

	First Lens			Second Lens		
	$\frac{1}{4} G^a_5 R_a R_2$	$-G^a_7 R_a \frac{1}{l'_2}$	$+G^a_8 R^2_a$	$\frac{1}{4} G^b_5 R_b R_3$	$-G^b_7 R_b \frac{1}{l_3}$	$-G^b_8 R^2_b$
$\log \frac{1}{4}$	9·3979			9·3979		
$+\log H'_k \cdot y^2$	9·3979	9·3979	9·3979	9·3979	9·3979	9·3979
$+\log G$	0·2348	9·8376*n*	9·5940	0·3037	9·9108*n*	9·7033*n*
$+\log R^n$	9·6783	9·6783	9·3566	9·3731*n*	9·3731*n*	8·7462
$+\log \left(\frac{1}{l'_2}\right)$		9·3922			9·3922	
log sum	8·7089	8·3060*n*	8·3485	8·4726*n*	8·0740	7·8474*n*
antilogs	0·0512R_2	−0·0202	0·0223	−0·0297R_3	0·0119	−0·0070

Collecting the terms, we get :—

Total $CC' = 0\cdot0512\,R_2 - 0\cdot0297\,R_3 + 0\cdot0070$.

And for a cemented objective, where R_3 will be equal to R_2,

Total $CC' = 0\cdot0215\,R_2 + 0\cdot0070$.

Substituting various curvatures (as before) for R_2, we find

For R_2 =	−0·10	−0·15	−0·20	−0·25	−0·30	−0·35	−0·40
CC' =	+0·0048	+0·0038	+0·0027	+0·0016	+0·0006	−0·0005	−0·0016

These points are then plotted against the contact curvature R_2 on the same graph as the spherical aberration, and the resultant coma curve (a straight line) will be obtained as indicated by the line marked No. 1 in the lower half of Fig. 14.

* N.B.—By choosing $H'_k = 1''$ we get a value for Coma which is comparable with the result obtained by the OSC' calculation (used in Chapter 1), but not necessarily identical; for the OSC' may show higher aberrational coma, whilst the $\Sigma\, CC'$ will be primary only.

The foregoing set of calculations must now be carried out in turn for the other pairs of glasses. The full working out will not be given here (although students should do this for themselves) but it may be helpful to give the final equations for the determination of the spherical aberration and coma for the cemented objective in each case:—

Glass Pair No. 2.

Spherical Aberration $(LA'p)$ $= 0{\cdot}468 + 0{\cdot}651\,R_2 + 7{\cdot}768\,R_2^2$ (crown)
$- 0{\cdot}160 + 1{\cdot}261\,R_3 - 4{\cdot}848\,R_3^2$ (flint)
Total Sph. Ab. $(LA'p)$ $= 0{\cdot}308 + 1{\cdot}912\,R_2 + 2{\cdot}920\,R_2^2$
Total coma contribution CC' $= 0{\cdot}0209\,R_2 + 0{\cdot}0067.$

Glass Pair No. 3.

Spherical Aberration $(LA'p)$ $= 0{\cdot}224 + 0{\cdot}591\,R_2 + 5{\cdot}838\,R_2^2$ (crown)
$- 0{\cdot}066 + 0{\cdot}813\,R_3 - 2{\cdot}766\,R_3^2$ (flint)
Total Sph. Ab. $(LA'p)$ $= 0{\cdot}158 + 1{\cdot}404\,R_2 + 3{\cdot}071\,R_2^2$
Total coma contribution CC' $= 0{\cdot}0216\,R_2 + 0{\cdot}0049.$

Glass Pair No. 4.

Spherical Aberration $(LA'p)$ $= 0{\cdot}310 + 0{\cdot}734\,R_2 + 6{\cdot}510\,R_2^2$ (crown)
$- 0{\cdot}087 + 0{\cdot}965\,R_3 - 3{\cdot}447\,R_3^2$ (flint)
Total Sph. Ab. $(LA'p)$ $= 0{\cdot}223 + 1{\cdot}699\,R_2 + 3{\cdot}063\,R_2^2.$
Total coma contribution CC' $= 0{\cdot}216\,R_2 + 0{\cdot}0059.$

From these equations and by substituting suitable values for R_2, we may obtain the necessary figures for plotting the spherical aberration and coma curves for each pair of glasses. These should all be plotted on one graph and given their corresponding pair number as shown in Fig. 14.

If we analyse these curves several interesting points emerge from them. Firstly, the spherical aberration plots as a parabola, and the coma as a straight line; secondly, the point at which the coma curve crosses the abscissa corresponds to the lowest part of the parabola in each case. Thirdly, it will be evident that by a suitable choice of glasses the parabola may be raised or lowered with respect to the abscissa, for example parabola No. 1 is much more below the axis than No. 3, whilst No. 4 and No. 2 are intermediate stages; consequently the ideal condition would be to arrange that the parabola just touched the abscissa at the same point that the coma curve crosses it, thus giving zero spherical aberration and zero coma. Fourthly, it will be noted that certain pairs of glasses will move the parabola sideways, e.g., No. 4 and No. 3 being more to the left than Nos. 1 and 2. The advantage of this is that the solution giving minimum spherical aberration and coma simultaneously can be made to give a longer radius r_2 for the contact surface thus tending to reduce zonal spherical aberration.

To face page 40.

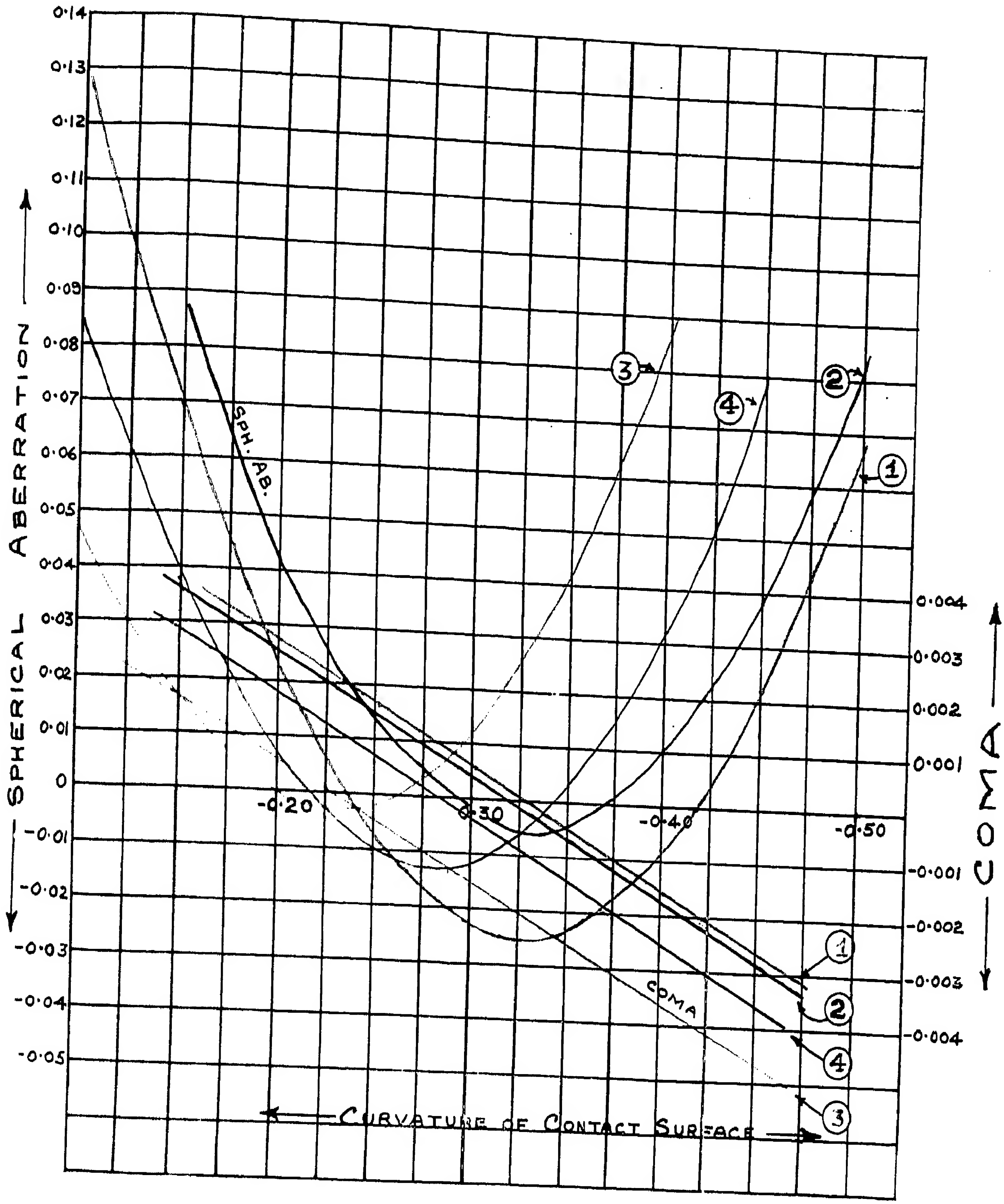


Fig. 14.

From the curves shown in Fig. 14 it will be evident that the best given by the four pairs of glasses chosen here would be that given No. 3 for the amounts of spherical aberration and coma are nearly z the contact curvature R_2 is $-0 \cdot 225$.

Therefore, taking this pair of glasses and this solution for R_2, we find from $R_a = R_1 - R_2$ and $R_b = R_3 - R_4$ that $R_1 = 0 \cdot 3894 + (-0 \cdot 225) = +0 \cdot 1644$ and $R_4 = -0 \cdot 225 - (-0 \cdot 1457) = -0 \cdot 0793$, so that

$$\left.\begin{aligned} r_1 &= +6 \cdot 083 \text{ inches} \\ r_2 \equiv r_3 &= -4 \cdot 444 \text{ inches} \\ r_4 &= -12 \cdot 610 \text{ inches} \end{aligned}\right\}.$$

From a scale drawing, a suitable axial thickness of the crown lens will be 0.200 inches and for the flint $0 \cdot 150$ inches.

With the glasses given for pair No. 3 in Table II the specification is now complete and we are ready to commence the trigonometrical ray-tracing test on the objective.

It is usual to test the chromatic aberration first, and this requires the refractive indices N_C and N_F. From the partial dispersions of these two glasses given in Table II we find for the crown :—

$$\begin{aligned} N_C &= N_D - (N_D - N_C) = 1 \cdot 51750 - 0 \cdot 00250 = 1 \cdot 51500 \\ \text{and } N_F &= N_D + (N_F - N_D) = 1 \cdot 51750 + 0 \cdot 00606 = 1 \cdot 52356 \end{aligned}$$

and for the flint :—

$$\begin{aligned} N_C &= N_D - (N_D - N_C) = 1 \cdot 69700 - 0 \cdot 00646 = 1 \cdot 69054 \\ \text{and } N_F &= N_D + (N_F - N_D) = 1 \cdot 69700 + 0 \cdot 01641 = 1 \cdot 71341 \end{aligned}$$

We also require Y, the height from the axis of the initial incident parallel ray, and in accordance with the usual practice for testing the chromatic aberration we obtain this from $Y = \sqrt{0 \cdot 5}$ of the lens semi-aperture, so that $Y = 0 \cdot 7071 \times 0 \cdot 5 = 0 \cdot 3536$.

The ray-trace is shown in Calculation No. 13.
Thus, the chromatic aberration $(L'_C - L'_F) = -0 \cdot 0039$ inches. The chromatic aberration tolerance is

$$\pm \frac{0 \cdot 5\lambda}{N' \,.\, \sin^2 U'_{0 \cdot 7071}} = \frac{0 \cdot 5 \times 0 \cdot 000022}{\sin^2 2^\circ\text{–}1'\text{–}2''}$$
$$= \pm 0 \cdot 0089 \text{ inches.}$$

It will be seen that the lens is fairly well corrected for colour, the chromatic aberration (over-corrected) being slightly less than half the permissible tolerance. In many cases, however, the value for the last radius as deduced from $R_b = R_3 - R_4$ will not give chromatic correction within the tolerance,

and consequently we need a correcting formula which will enable this to be done. A formula which involves the use of the intersection lengths of the rays after they have passed through the last surface is Chr. (3), namely :—

$$\frac{1}{\text{new } r} = \frac{1}{r_{\substack{\textit{already}\\ \textit{used.}}}} + \left\{ \frac{(L'_C - L'_F)_{\substack{\textit{already}\\ \textit{used.}}} - (L'_C - L'_F)_{\textit{prescribed}}}{(N_F - N_C)_{\substack{\textit{flint}\\ \textit{glass.}}} \times (L'_C \,.\, L'_F)_{\substack{\textit{already}\\ \textit{used.}}}} \right\}$$

and if we apply this to the present example we find

$$\frac{1}{\text{new } r} = \frac{1}{-12\cdot610} + \left\{ \frac{-0\cdot0039 - 0\cdot0000}{0\cdot02289 \times 9\cdot8926 \times 9\cdot8965} \right\}$$

$$\therefore \text{ new } r_4 = -12\cdot340 \text{ inches.}$$

If C and F rays from the contact surface are now traced through surface 4 using this radius, it will be found that $L'_C = 9\cdot7764$ and $L'_F = 9\cdot7766$; giving a chromatic aberration of $-0\cdot0002$ inches.

In order to save the extra work of such an additional ray-trace, it is better to apply a correcting formula Chr. (1) *immediately after the ray-trace through the contact surface*, and then to use the newly determined r_4 (by this formula) straight away without troubling about the use of the r_4 obtained from $R_b = R_3 - R_4$.

Chr. (1) is :— $$\frac{1}{r} = \frac{(L_C - L_F)\,N_C}{L_C \,.\, L_F(N_F - N_C)} + \frac{1}{L_F}$$

In our calculation No. 13, we have $L_{4C} = +36\cdot6239$ and $L_{4F} = 38\cdot6775$, and the mean dispersion $(N_F - N_C)$ for the flint glass employed is $0\cdot02287$, so that :—

$$\frac{1}{r} = \frac{(36\cdot6239 - 38\cdot6775) \times 1\cdot69054}{36\cdot6239 \times 38\cdot6775 \times 0\cdot02287} + \frac{1}{38\cdot6775} = -0\cdot08131$$

$$\therefore \quad r_4 = -12\cdot299 \text{ inches.}$$

Tracing through the last surface with the above radius (see calculation No. 14) we find the chromatic aberration comes out at $0\cdot0000$ inches.

Having corrected the chromatic aberration—in this case perfectly—the spherical aberration will now be tested trigonometrically. The data for this will be :—

$$\left.\begin{aligned} r_1 &= +6\cdot083 \text{ inches} \\ r_2 \equiv r_3 &= -4\cdot444 \text{ inches} \\ r_4 &= -12\cdot299 \text{ inches} \end{aligned}\right\} \quad \begin{aligned} d'_1 &= 0\cdot200 \text{ inches} \\ d'_2 &= 0\cdot150 \text{ inches} \end{aligned} \qquad Y = 0\cdot500$$

and the refractive indices for "brightest light" are obtained from $N_D + 0\cdot188\,(N_F - N_C)$ for each glass, namely $1\cdot51911$ and $1\cdot70130$ for crown

Calculation No. 13

$Y = 0{\cdot}3536$ $r_1 = +6{\cdot}083''$, $r_2 \equiv r_3 = -4{\cdot}444''$, $r_4 = -12{\cdot}610''$ $d_1' = 0{\cdot}200''$, $d_2' = 0{\cdot}150''$

	Crown	*Flint*
$N_C =$	1·51500	1·69054
$N_F =$	1·52356	1·71341

	First Surface		*Second Surface*		*Third Su*	
	C ray	*F* ray	*C* ray	*F* ray	*C* ray	
L $-r$			17·6797 + 4·444	17·4889 + 4·444	36·6239 +12·610	
$(L-r)$.	$Y = 0{\cdot}3536$		22·1237	21·9329	49·2339	
log sin U + log $(L-r)$			8·29631 1·34486	8·30097 1·34110	7·97849 1·69226	
log $(L-r)$ sin U − log r	9·54851 0·78412	9·54851 0·78412	9·64117 0·64777*n*	9·64207 0·64777*n*	9·67075 1·10072*n*	
log sin I + log $\frac{N}{N'}$	8·76439 9·81959	8·76439 9·81714	8·99340*n* 9·95239	8·99430*n* 9·94900	8·57003*n* 0·22802	
log sin I' + log r	8·58398 0·78412	8·58153 0·78412	8·94579*n* 0·64777*n*	8·94330*n* 0·64777*n*	8·79805*n* 1·10072*n*	
log r . sin I' − log sin U'	9·36810 8·29631	9·36565 8·30097	9·59356 7·97849	9·59107 7·95485	9·89877 8·54654	
log $(L'-r)$	1·07179	1·06468	1·61507	1·63622	1·35223	
U $+I$	0– 0– 0 3–19–57	0– 0– 0 3–19–57	1– 8– 1 – 5–39– 8	1– 8–45 – 5–39–51	0–32–43 – 2– 7–46	 -
$U+I$ $-I'$	3–19–57 2–11–56	3–19–57 2–11–12	– 4–31– 7 5– 3–50	– 4–31 –6 5– 2– 5	– 1–35– 3 3–36– 5	-
U'	1– 8– 1	1– 8–45	0–32–43	0–30–59	2– 1– 2	
$L'-r$ $+r$	11·7975 6·083	11·6059 6·083	41·2164 – 4·444	43·2733 – 4·444	22·5025 – 12·610	 -
L' $-d'$	17·8805 0·200	17·6889 0·200	36·7724 0·150	38·8293 0·150	9·8925	
new L	17·6805	17·4889	36·6224	38·6793		
$-\frac{1}{2}U$ $+\frac{1}{2}I$	0– 0– 0 1–39–58	0– 0– 0 1–39–58	– 0–34– 0 – 2–49–34	– 0–34–22 – 2–49–56	– 0–16–22 – 1– 3–53	- -
$\frac{1}{2}(I-U)$	1–39–58	1–39–58	– 3–23–34	– 3–24–18	– 1–20–15	-
$\frac{1}{2}I'$ $-\frac{1}{2}U'$	1– 5–58 – 0–34– 0	1– 5–36 – 0–34–22	– 2–31–55 – 0–16–22	– 2–31– 2 – 0–15–30	– 1–48– 2 – 1– 0–31	- -
$\frac{1}{2}(I'-U')$	0–31–58	0–31–14	– 2–48–17	– 2–46–32	– 2–48–33	-
log L + log sin U or log Y (for // light) + log sec $\frac{1}{2}(I-U)$	 9·54851 0·00018	 9·54851 0·00018	1·24748 8·29631 0·00076	1·24276 8·30097 0·00077	1·56376 7·97849 0·00012	
log PA + log cosec U' + log cos $\frac{1}{2}(I'-U')$	9·54869 1·70369 9·99998	9·54869 1·69903 9·99998	9·54455 2·02151 9·99948	9·54450 2·04515 9·99949	9·54237 1·45346 9·99948	
log L'	1·25236	1·24770	1·56554	1·58914	0·99531	[illegible]
L' (by check)	17·8797	17·6889	[illegible]	[illegible]	[illegible]	

CALCULATION No. 14

Ray-trace through last surface corrected by Chr. (1), i.e., $r_4 = -12 \cdot 999$ inches.

LAST SURFACE

	C ray	F ray
L $-r$	36·6239 +12·299	38·6775 +12·299
$(L-r)$	48·9229	50·9765
log sin U + log $(L-r)$	7·97849 1·68951	7·95485 1·70737
log $(L-r)$ sin U − log r	9·66800 1·08987n	9·66222 1·08987n
log sin I + log $\frac{N}{N'}$	8·57813n 0·22802	8·57235n 0·23386
log sin I' + log r	8·80615n 1·08987n	8·80621n 1·08987n
log r . sin I' − log sin U'	9·89602 8·55248	9·89608 8·55254
log $(L'-r)$	1·34354	1·34354
U + I	0–32–43 − 2–10–10	0–30–59 − 2– 8–27
$U+I$ − I'	− 1–37–27 3–40– 9	− 1–37–28 3–40–11
U'	2– 2–42	2– 2–43
$L'-r$ + r	22·0567 − 12·299	22·0567 − 12·299
final L'	9·7577	9·7577
$-\frac{1}{2}U$ $+\frac{1}{2}I$	− 0–16–22 − 1– 5– 5	− 0–15–30 − 1– 4–14
$\frac{1}{2}(I-U)$	− 1–21–27	− 1–19–44
$\frac{1}{2}I'$ $-\frac{1}{2}U'$	− 1–50– 4 − 1– 1–21	− 1–50– 6 − 1– 1–22
$\frac{1}{2}(I'-U')$	− 2–51–25	− 2–51–28
log L + log sin U + log sec $\frac{1}{2}(I-U)$	1·56376 7·97849 0·00012	1·58746 7·95485 0·00012
log PA + log cosec U' + log cos $\frac{1}{2}(I'-U')$	9·54237 1·44752 9·99946	9·54243 1·44746 9·99946
log L'	0·98935	0·98935
final L' (by check)	9·7578	9·7578

Chromatic Aberration $(L'_C - L'_F) = 0 \cdot 0000$ inches.

Chrom. Ab. Tolerance $= \pm \dfrac{0 \cdot 5\ \lambda}{N' \sin^2 U'_{0 \cdot 7071}} = \pm\ 0 \cdot 0089$ inches.

and flint respectively. Calculation No. 15 shows the ray tracing for a marginal and paraxial ray.

It will be seen from the resulting figures that the spherical aberration is well corrected, being less than one twentieth of the permissible tolerance.

It now only remains to determine the amount of coma, and in order to obtain this accurately it would be necessary to trace three oblique rays through the lens at a given H'_k (see Fig. 13) which shows tangential coma approximately proportional to the sagittal coma. But as oblique ray-tracing has not been dealt with at this stage, we may apply an analytical formula for the Offence against the Sine Condition (OSC'). Now $OSC' = \dfrac{\text{coma}}{H'_k}$ very closely (see foot-note on page 39), and as we have chosen H'_k to be one inch, the value obtained for OSC' will be equal to that for the coma.

From Sine Theorem I (Conrady's *Applied Optics*, pp. 367–370) the

$$OSC' = 1 - \left(\frac{\sin U_1}{u_1}\right) \cdot \frac{u'_k}{\sin U'_k} \cdot \frac{l'_k - l'pr_k}{L'_k - l'pr_k}$$

where $l'pr_k$ is the distance of the stop or diaphragm from the lens system. (In the case of a telescope objective the stop is assumed to be *on* the lens and therefore $l'pr_k$ will be zero.)

For incident parallel light the quantity $\left(\dfrac{\sin U_1}{u_1}\right)$ becomes $\left(\dfrac{Y}{y}\right)$ and as our normal practice is to make $y = Y$, this term is unity and can be omitted.

Thus $OSC' = 1 - \dfrac{u'_k \cdot l'_k}{\sin U'_k \cdot L'_k}$ and from Calculation No. 15 $u'_k = 0\cdot050478$; $U'_k = 2°–53'–35''$; $l'_k = 9\cdot7548$; and $L'_k = 9\cdot7564$, so that, inserting these figures in the foregoing equation it will be found that $OSC' = +\,0\cdot00005$.

This is considerably below the OSC' tolerance of $\pm\,0\cdot0025$ generally permissible for ordinary telescope objectives (see page 32); and indeed it is even below the more strict tolerance given by the Rayleigh limit of

$$\pm \frac{\frac{1}{2}\lambda}{N' \cdot H'_k \cdot \sin U'_m} = \pm\, 0\cdot00022.$$

Thus the design of this objective has proved to be an extremely satisfactory one, all the aberrations being well below the tolerances; and if made up to the specification given by the design, should produce an achromatic objective of excellent performance.

Further Points in the Design of Telescope Objectives

In the foregoing pages dealing with the systematic design of telescope objectives, it was seen that after the spherical aberration and coma equations had been obtained, curves were plotted for these aberrations against various lens "shapes". It is certainly better to do this graphically to begin with, for a mental picture as to the rate of change of these two aberrations simultaneously is readily obtained and the best compromise solution more easily arrived at. For a cemented doublet, however, the spherical aberration equation is a quadratic in R_2 and can be solved independently for any desired value of the spherical aberration (including zero, of course), this being done numerically without resorting to the graphical plotting. As an example of the latter case, let us take a telescope objective of the same previous requirements but made from a crown glass with $N_D = 1 \cdot 5407$ and a flint glass with $N_D = 1 \cdot 6225$.* By the analytical G-sum calculations (already described) we find that the spherical aberration contribution equations for the crown and flint lenses are :—

$$LA' = 0 \cdot 680 + 1 \cdot 258\, R_2 + 11 \cdot 960\, R_2^2 \text{ (crown)}$$

and $$LA' = -0 \cdot 207 + 1 \cdot 874\, R_3 - 7 \cdot 030\, R_3^2 \text{ (flint)}$$

giving a cemented objective (where $R_2 = R_3$) a total contribution of

$$LA' = 0 \cdot 473 + 3 \cdot 132\, R_2 + 4 \cdot 930\, R_2^2.$$

Suppose we wish in this case to have our objective completely free from spherical aberration, we then have :—

$$4 \cdot 930\, R_2^2 + 3 \cdot 132\, R_2 + 0 \cdot 473 = 0$$

and solving this quadratic equation in the usual way

$$R_2 = -\frac{3 \cdot 132}{9 \cdot 86} \pm \sqrt{\left(\frac{3 \cdot 132}{9 \cdot 86}\right)^2 - \frac{0 \cdot 473}{4 \cdot 930}} = -0 \cdot 3180 \pm 0 \cdot 0714.$$

Thus R_2 will be either $-0 \cdot 3894$ or $-0 \cdot 2466$.

As the total curvatures $\mathcal{R}_a$ and $\mathcal{R}_b$ for the crown and flint components for this lens were $+0 \cdot 4700$ and $-0 \cdot 2470$ respectively, and taking the first solution for $R_2 = -0 \cdot 3894$, we find

$$R_1 = \mathcal{R}_a + R_2 = 0 \cdot 4700 - 0 \cdot 3894 = +0 \cdot 0806$$

and $$R_4 = R_2 - \mathcal{R}_b = -0 \cdot 3894 + 0 \cdot 2470 = -0 \cdot 1424.$$

Therefore, $r_1 = +12 \cdot 407''$; $r_2 \equiv r_3 = -2 \cdot 568''$; and $r_4 = -7 \cdot 022''$.

(N.B.—When corrected for complete achromatism by ray-tracing r_4 is found to be $-6 \cdot 766''$ or $R_4 = -0 \cdot 1478$).

The above specification, according to the analytical solution, should give freedom from spherical aberration.

* The optical constants of these glasses are given in Chapter I, page 23.

When tested trigonometrically, however, there is a residual of $+0 \cdot 0057''$ outstanding.

Correction of Spherical Aberration

An example will now be taken to illustrate the way in which this residual may be corrected or at least reduced to within the permissible tolerance.

As mentioned earlier, spherical aberration may be altered by " bending " the lens-system as a whole, but in order to avoid a series of haphazard trial bendings a differential method can be used for calculating the change in curvature δR_2 necessary to produce the desired state of spherical correction —in this case zero.

The change in curvature $\delta R_2 = \dfrac{(\text{Sph. Ab.}_{required} - \text{Sph. Ab.}_{found})}{\dfrac{d\,.\,LA'}{d\,.\,R_2}}$ where $\dfrac{d\,.\,LA'}{d\,.\,R_2}$ is the required differential co-efficient. This latter may be obtained by using the original analytical equation on which the solution was based, namely

$$LA' = 0 \cdot 473 + 3 \cdot 132\, R_2 + 4 \cdot 930\, R_2^2.$$

And the differential co-efficient therefore will be :—

$$\frac{d\,.\,LA'}{d\,.\,R_2} = 3 \cdot 132 + 9 \cdot 860\, R_2$$

and substituting the value $-0 \cdot 3894$ for R_2 we find

$$\frac{d\,.\,LA'}{d\,.\,R_2} = -0 \cdot 704.$$

As $LA'_{required}$ is zero and LA'_{found} (by ray-tracing) was $+0 \cdot 0057$

$$\delta R_2 = \frac{(0 - 0 \cdot 0057)}{-0 \cdot 704} = +0 \cdot 0081$$

The previous curvatures were $R_1 = 0 \cdot 0806$; $R_2 \equiv R_3 = -0 \cdot 3894$ and R_4 (corrected) $= -0 \cdot 1478$; so that if we add algebraically δR_2 to each of these, we get the new $R_1 = 0 \cdot 0887$; $R_2 \equiv R_3 = -0 \cdot 3813$; and $R_4 = -0 \cdot 1397$; and the corresponding radii would be $r_1 = +11 \cdot 274''$, $r_2 \equiv r_3 = -2 \cdot 622''$; and $r_4 = -7 \cdot 158''$.

If a ray-tracing test is now made with these new radii it will be found that the spherical aberration is then $-0 \cdot 0008''$, thus indicating that a substantial reduction in the previous residual has been secured and a value which is well below the permissible tolerance.

Non-cemented or Broken-Contact Objectives

Having dealt with objectives which have the same contact radius, we will now consider the advantages or disadvantages of an objective in which the contact surfaces are not similar. Obviously an additional degree of freedom in the design is afforded by the latter case, and this leads to the fact that both spherical aberration and coma can be corrected simultaneously ; but from the glass workshop's point of view this entails an additional pair of grinding and polishing tools which is a slight disadvantage, however. Returning to the question of the design, let us take the equations for the

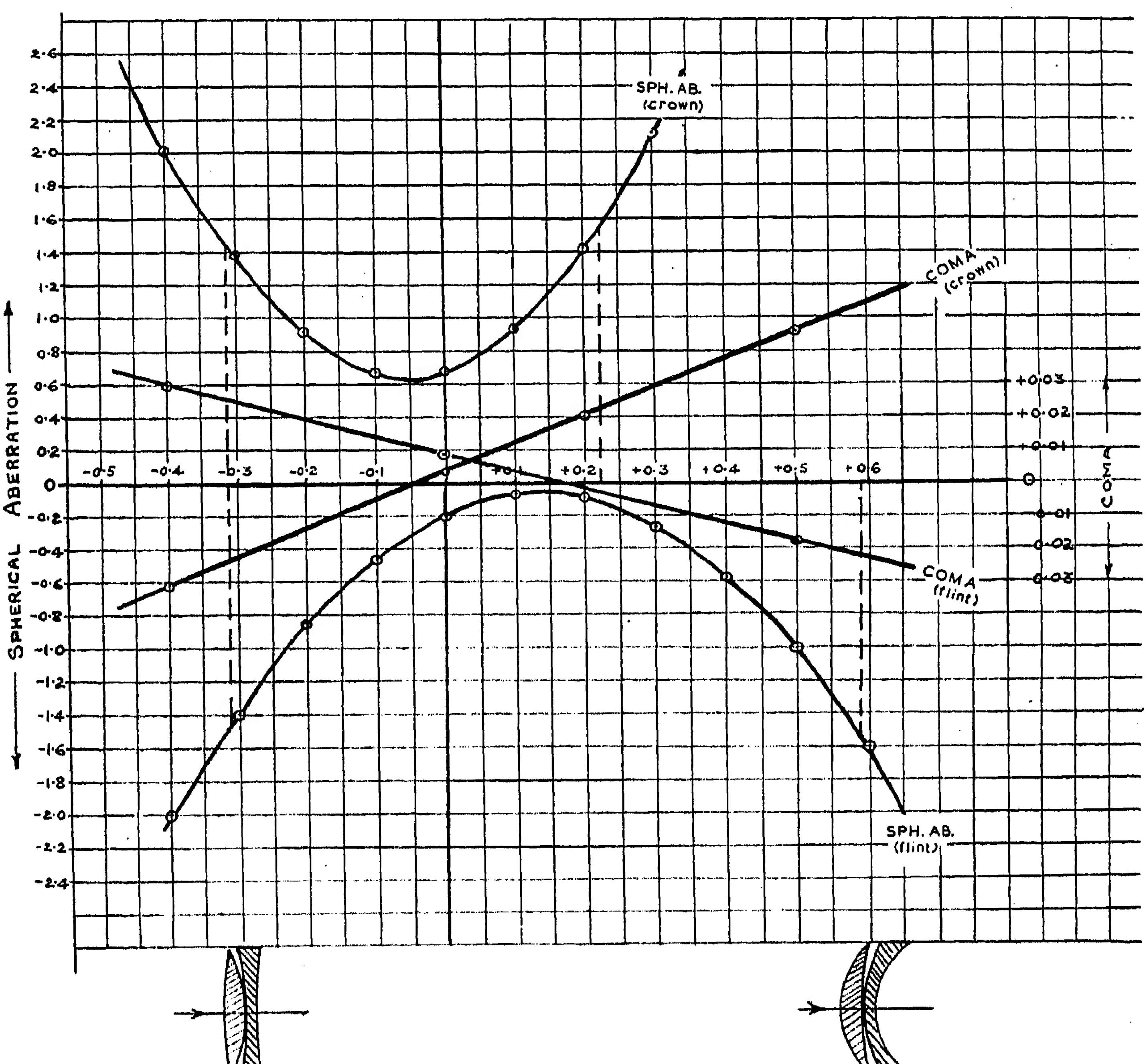


Fig. 15.

spherical aberration and coma for each of the two components as in the previous example, namely :—

Sph. Ab. $\begin{cases} LA' = 0\cdot680 + 1\cdot258\,R_2 + 11\cdot960\,R_2^2 \text{ (crown)} \\ LA' = -0\cdot207 + 1\cdot874\,R_3 - 7\cdot030\,R_3^2 \text{ (flint)} \end{cases}$

Coma $\begin{cases} CC' = 0\cdot0858\,R_2 + 0\cdot0027 \text{ (crown)} \\ CC' = -0\cdot0509\,R_3 + 0\cdot0083 \text{ (flint)} \end{cases}$

and substituting suitable values for R_2 and R_3 the curves shown in Fig. 15 may be plotted. If then we look along the abscissa we find that at position $R_2 = +0\cdot224$, the spherical aberration is $+1\cdot570$ and the coma $+0\cdot022$ for the crown lens whilst at position $R_3 = +0\cdot592$ the spherical aberration amounts to $-1\cdot570$ and the coma $-0\cdot022$ for the flint lens, thus giving cancellation of these two aberrations simultaneously. With $R_a = +0\cdot470$ and $R_b = -0\cdot247$ as before, the corresponding values of R_1, R_2, R_3 and R_4 may be obtained from which the radii will be found to be as follows :— $r_1 = +1\cdot441''$; $r_2 = +4\cdot464''$; $r_3 = +1\cdot689''$; and $r_4 = +1\cdot192''$. This results in a shape of lens indicated diagramatically to the right of Fig. 15.

The other possible solution is when $R_2 = -0\cdot312$ and $R_3 = -0\cdot308$ when it will be seen from the graph that for the crown lens the spherical aberration value is $+1\cdot450$ and the coma $-0\cdot024$, whilst for the flint lens the spherical aberration is $-1\cdot450$ and the coma $+0\cdot024$, again giving elimination of these two aberrations simultaneously. The shape of this lens will be as to the left of Fig. 15 having radii of $r_1 = +6\cdot329''$; $r_2 = -3\cdot205''$; $r_3 = -3\cdot247''$; and $r_4 = -16\cdot393''$.

It will be observed from these two solutions that the second is a much more suitable one to produce from a manufacturing view point; also that the two contact surfaces in the second solution have very nearly the same radius whereas in the first they are widely different.

If the liberty is taken of making the contact surfaces of solution 2 identical, then a close approximation to a correction of both aberrations is secured (although not a strictly accurate correction) and this fact accounts for the predominance of cemented objectives with similar contact radii over the broken-contact type for a large number of purposes. But it must be remembered that this is not the ideal solution to the problem.

Numerical Method for Solving the Foregoing Solutions

Apart from the graphical method just described it is possible to find, mathematically, the best shape of the lenses which will give simultaneous correction of the spherical aberration and the coma. This may be done as follows :—

The coma contribution of the crown lens a, is

$$CC'_a = 0\cdot0858\,R_2 + 0\cdot0027$$

and the coma contribution of the flint lens b, is

$$CC'_b = -0\cdot0509\,R_3 + 0\cdot0083.$$

If $CC'_a + CC'_b$ is to be zero, we get by adding the equations

$$0\cdot0858\,R_2 - 0\cdot0509\,R_3 + 0\cdot0110 = 0$$

$$\therefore\quad R_3 = 1\cdot68\,R_2 + 0\cdot216 \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

This relation between R_2 and R_3 will ensure absence of coma.

Substituting this value of R_3 in the previous expression for LA'_b, namely

$$LA'_b = -0\cdot207 + 1\cdot874\,R_3 - 7\cdot030\,R_3^2$$

LA'_b now becomes $-19\cdot84\,R_2^2 - 1\cdot956\,R_2 - 0\cdot130$ . . . . . (ii)

We had previously

$$LA'_a = 0\cdot680 + 1\cdot258\,R_2 + 11\cdot960\,R_2^2 \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

And as the sum of the aberrations is to be zero, then we get, on adding (ii) and (iii),

$$0 = 7\cdot880\,R_2^2 + 0\cdot698\,R_2 - 0\cdot550$$

Solving this quadratic,

$$R_2 = \frac{-\ 0\cdot698}{15\cdot76} \pm \sqrt{\left(\frac{0\cdot698}{15\cdot76}\right)^2 - \frac{-\ 0\cdot550}{7\cdot88}}$$

$$= -\,0\cdot0443 \pm \sqrt{0\cdot07176}$$

$$\therefore\ R_2 = +\,0\cdot224 \text{ or } -\,0\cdot312.$$

So that with $\mathcal{R}_a = +\,0\cdot470$ and $\mathcal{R}_b = -\,0\cdot247$ we get :—

1st solution	*2nd solution*
$R_1 = \mathcal{R}_a + R_2 = 0\cdot470 + 0\cdot224$	$R_1 = \mathcal{R}_a + R_2 = 0\cdot470 - 0\cdot312$
$= 0\cdot694$	$= +\,0\cdot158$
$\therefore\ r_1 = +\,1\cdot441''$	$\therefore\ r_1 = +\,6\cdot329''$
$R_2 = +\,0\cdot224\ ;\ \therefore\ r_2 = +\,4\cdot464''$	$R_2 = -\,0\cdot312\ ;\ \therefore\ r_2 = -\,3\cdot205''$

Substituting the two above values of R_2 in turn in equation (i)

$R_3 = +\,0\cdot592\ ;\ \therefore\ r_3 = +\,1\cdot689''$ and $R_3 = -\,0\cdot308\ ;\ \therefore\ r_3 = -\,3.247''$

From $R_4 = R_3 - \mathcal{R}_b$

$R_4 = 0\cdot592 + 0\cdot247 = +\,0\cdot839$	$R_4 = -\,0\cdot308 + 0\cdot247 = -\,0\cdot061$
$\therefore\ r_4 = +\,1\cdot192''$	$\therefore\ r_4 = -\,16\cdot393''$

Triple-Lens Telescope Objectives

In the foregoing examples in the design of a telescope objective, two glasses only were employed and the chromatic aberration was corrected for two wavelengths, namely for the red (C) and blue (F) rays. If, however, a

better state of chromatic aberration is required with a consequent reduction in the secondary spectrum, it is necessary to employ three glasses and correct the lens for three colours. An example will be given to illustrate the procedure in this case. Perusal of pages 159 to 166 of Conrady's *Applied Optics* will be found advantageous to this numerical illustration.

Let us assume that we require an objective of similar focal length and aperture as in the previous section, namely ten inches and one inch respectively, and that the glasses to be used are as follows :—

Lens	*Glass*		*Mean Dispersion*		*Partial Dispersions*			*Ratios of Partial Dispersions to mean Dispersions*	
		N_D	$(N_F - N_C) = \delta N$	V	$N_D - N_C$	$N_F - N_D$	$N_{G'} - N_F$	$\dfrac{N_{G'} - N_F}{N_F - N_C}$	$\dfrac{N_F - N_D}{N_F - N_C}$
a	BSC	1·5160	0·00809	63·8	0·00242	0·00567	0·00454	0·561	0·701
b	LBF	1·5376	0·01069	50·3	0·00313	0·00756	0·00616	0·576	0·707
c	LBF	1·5833	0·01251	46·6	0·00362	0·00889	0·00740	0·592	0·711

The first thing to be done is to find the total curvatures R_a, R_b and R_c of the lenses of the combination from :—

$$R_a = -\frac{1}{f'} \cdot \frac{1}{\delta P_b} \cdot \frac{1}{V_a - V_c} \cdot \frac{1}{\delta N_a} (P_c - P_b)$$

$$R_b = +\frac{1}{f'} \cdot \frac{1}{\delta P_b} \cdot \frac{1}{V_a - V_c} \cdot \frac{1}{\delta N_b} (P_c - P_a)$$

$$R_c = -\frac{1}{f'} \cdot \frac{1}{\delta P_b} \cdot \frac{1}{V_a - V_c} \cdot \frac{1}{\delta N_c} (P_b - P_c)$$

where the P values are the ratio values given in the last column but one (in the above table) divided by those in the last column ; in other words

$P = \left(\dfrac{N_{G'} - N_F}{N_F - N_D}\right)$ for each glass, for example

$$P_a = \frac{0\cdot561}{0\cdot701} = 0\cdot8003\ ; \quad P_b = \frac{0\cdot576}{0\cdot707} = 0\cdot8147\ ; \quad \text{and} \quad P_c = \frac{0\cdot592}{0\cdot711} = 0\cdot8326.$$

The quantity $\delta P_b = (P_c - P_a)\dfrac{V_b - V_c}{V_a - V_c} - (P_c - P_b)$

$$\therefore\ \delta P_b = 0\cdot0323 \left(\frac{3\cdot7}{17\cdot2}\right) - (0\cdot0179) = -0\cdot0110$$

so that, $$R_a = \frac{1}{-10 \times -0\cdot011 \times 17\cdot2 \times 0\cdot00809} \times \frac{0\cdot0179}{1} = +1\cdot171$$

$$\mathcal{R}_b = \frac{1}{10 \times -0 \cdot 011 \times 17 \cdot 2 \times 0 \cdot 01069} \times \frac{0 \cdot 0323}{1} = -1 \cdot 597$$

$$\mathcal{R}_c = \frac{1}{-10 \times -0 \cdot 011 \times 17 \cdot 2 \times 0 \cdot 01251} \times \frac{0 \cdot 0144}{1} = +0 \cdot 608(3)$$

Having obtained the curvatures of the three component lenses, we can "bend" them to any convenient shape and choose a solution which looks promising and at the same time involves the least number of tools in the glass workshop.

Taking first then the choice of making lenses a and c equi-convex, and lens b equi-concave, we have :—

$\mathcal{R}_a = R_1 - R_2$; $\therefore$ $R_1 = \frac{\mathcal{R}_a}{2} = \frac{1 \cdot 171}{2} = 0 \cdot 5855$; giving $r_1 = +1 \cdot 708''$
and $r_2 = -1 \cdot 708''$

$\mathcal{R}_b = R_3 - R_4$; $\therefore$ $R_3 = \frac{\mathcal{R}_b}{2} = -0 \cdot 7985$; giving $r_3 = -1 \cdot 252''$
and $r_4 = +1 \cdot 252''$

$\mathcal{R}_c = R_5 - R_6$; $\therefore$ $R_5 = \frac{\mathcal{R}_c}{2} = +0 \cdot 3042$; giving $r_5 = +3 \cdot 287''$
and $r_6 = -3 \cdot 287''$

This lens is drawn in Fig. 16a, in which it will be noted there are two thin "air-lenses".

As a second choice, let us make the front lens equi-convex as before but have a cemented contact between lens a and b (i.e., $r_3 \equiv r_2$). Also let us make the last radius of the third lens equal to ∞ (i.e., a flat surface); then

$$r_1 = +1 \cdot 708''$$
$$r_2 = -1 \cdot 708''$$
$$r_3 = -1 \cdot 708''$$

$\mathcal{R}_b = R_3 - R_4$

$-1 \cdot 597 = -0 \cdot 5855 - R_4$

$\therefore$ $R_4 = +1 \cdot 012$, so that $r_4 = +0 \cdot 989''$

$\mathcal{R}_c = R_5 - R_6$

$0 \cdot 6083 = R_5 - 0$ so that $r_5 = +1 \cdot 644$ and $r_6 = \infty$

Fig. 16b shows this shape of the lens, which now has only one air-space.

And thirdly, let us choose a shape which gives an all-cemented lens combination with a flat last surface.

Then $r_6 = \infty$; $r_5 = +1 \cdot 644''$; $r_4 = +1 \cdot 644''$

$\mathcal{R}_b = R_3 - R_4$ (i.e.) $-1 \cdot 597 = R_3 - 0 \cdot 608$; $\therefore$ $R_3 = -0 \cdot 989$; $r_3 = -1 \cdot 011''$
and $r_2 = -1 \cdot 011''$

$\mathcal{R}_a = R_1 - R_2$ (i.e.) $1 \cdot 171 = R_1 + 0 \cdot 989$; $\therefore$ $R_1 = 0 \cdot 182$; $r_1 = +5.495''$

This form of the triple lens is shown in Fig. 16c and as it is probably the least difficult of the three to make in the glass workshop, let us take this lens and test it trigonometrically in order to see what the chromatic correction for three colours is like.

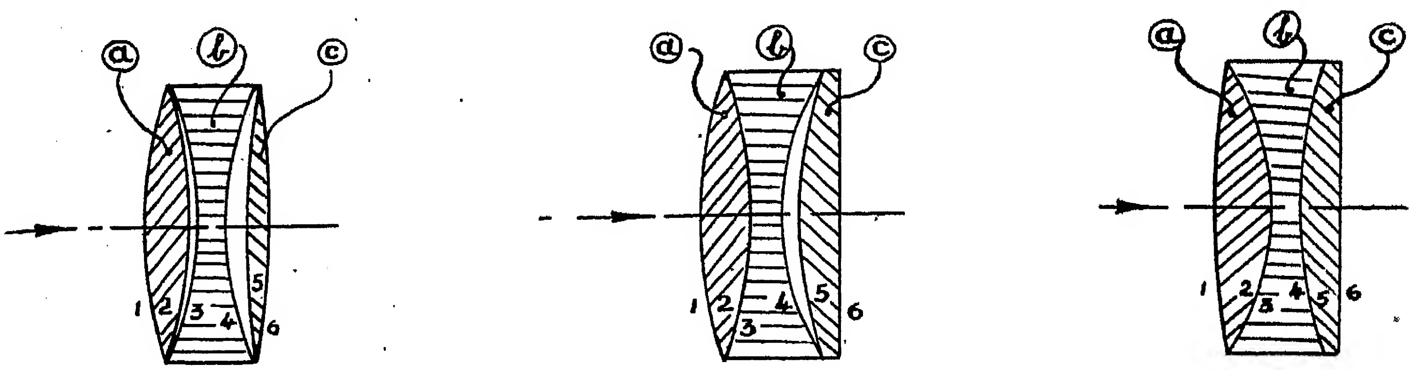


Fig. 16.

To do this we must trace C, F and G′ rays through the complete lens system, starting with an initial "white" ray at $\sqrt{0 \cdot 5}$ of the semi-aperture.

The complete specification for such a calculation will therefore be :—

Radii	*Axial thicknesses*	
$r_1 = +5 \cdot 495''$		
$r_2 = -1 \cdot 011''$	$d_1' = 0 \cdot 17''$	
$r_3 = -1 \cdot 011''$		$Y = 0 \cdot 3536''$
$r_4 = +1 \cdot 644''$	$d_3' = 0 \cdot 10''$	
$r_5 = +1 \cdot 644''$		
$r_6 = \infty$	$d_5' = 0 \cdot 13''$	

The refractive indices will be :—

	Lens a	*Lens b*	*Lens c*
N_C	1·51358	1·53447	1·57968
N_F	1·52167	1·54516	1·59219
N_G'	1·52621	1·55132	1·59959

If this ray-tracing is carried out the residuals of chromatic aberration will be found to be :—

$$(L_C' - L_F') = -0 \cdot 009''$$
$$(L_F' - L_{G'}') = -0 \cdot 014''$$
$$(L_C' - L_{G'}') = -0 \cdot 023''$$

The chromatic aberration tolerance (taking $\lambda = 4861.A$) is

$$\frac{0 \cdot 5\lambda}{N' \sin^2 U'_{0 \cdot 7071}} = \pm 0 \cdot 008''$$

It will be seen that the residuals are all outside the permissible tolerance, but it must be remembered that no correction (by adjustment of the last surface) has yet been applied. If this is done by using the formula Chr. (1) and utilizing the intersection lengths for C and G′ light in this formula, the corrected last radius will be found to be $-64 \cdot 808$ inches. Tracing through the last surface with this radius the following values will be found :—

$$(L'_C - L'_F) = +0 \cdot 0028$$
$$(L'_F - L'_{G'}) = -0 \cdot 0024 \qquad \text{Chrom. Ab. Tolerance} = \pm 0 \cdot 0065''$$
$$(L'_C - L'_{G'}) = +0 \cdot 0004$$

Thus, the lens is now well corrected for three colours of the spectrum. The spherical aberration and the coma are not determined here, but those who are interested will no doubt wish to go on with the determination of these in order to see how the design turns out as a whole. The example given here is merely to show the procedure for achromatization.

EYEPIECE DESIGN

THE simpler eyepieces employed in optical instruments, namely the Huygenian, the Ramsden and the four-lens terrestrial eyepiece are, with rare exceptions, made up of ordinary plano-convex crown glass lenses, but are nevertheless capable of giving perfectly satisfactory images when the best use is made of the restricted liberties for varying the design.

We may profitably begin with a few general observations on the chief characteristics of the two principal types and on their particular requirements. Fig. 17a shows the Huygenian type in which both plano-convex lenses have their plane surfaces facing towards the eye of the observer ; the field lens intercepts the converging rays from the objective before they come to a focus and produces a diminished real image *between* the two lenses, and this image is then viewed under magnification by the eye positioned behind the eye lens.

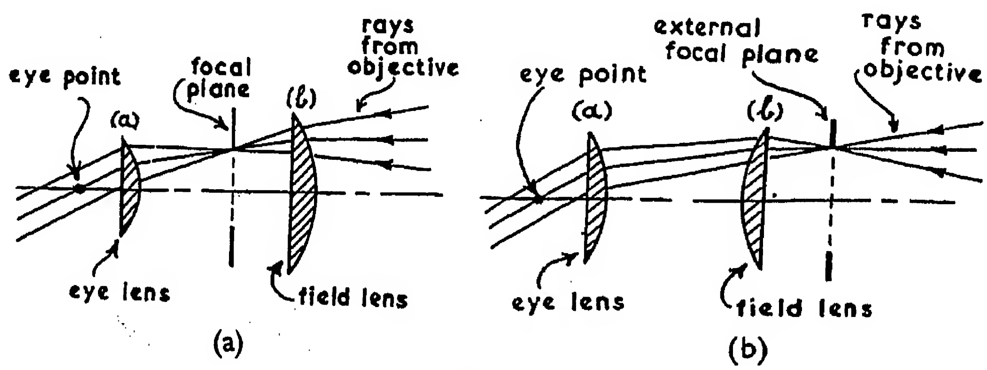


Fig. 17.

In the Ramsden eyepiece (Fig. 17b) the two convex surfaces are placed innermost and the lenses are of such a focal length that the focal plane of the eyepiece is situated just in front of the field lens where the image from the objective would also be formed.

In all these eyepieces, the disposition of the plano-convex lenses (namely, whether the plane surface is towards the eye of the observer or remote from it) is decided by the resulting oblique aberrations, more especially by the astigmatism and curvature of field. It can be shown by studying these oblique aberrations that the reduction of astigmatism and the flattening of the field can only be secured by using the eye lens with its flat surface towards the eye of the observer, and hence this is the orthodox practice.

Another detail of some importance is the position of the eye-point or exit-pupil, which marks the position at which an image of the objective is

To face page 55.

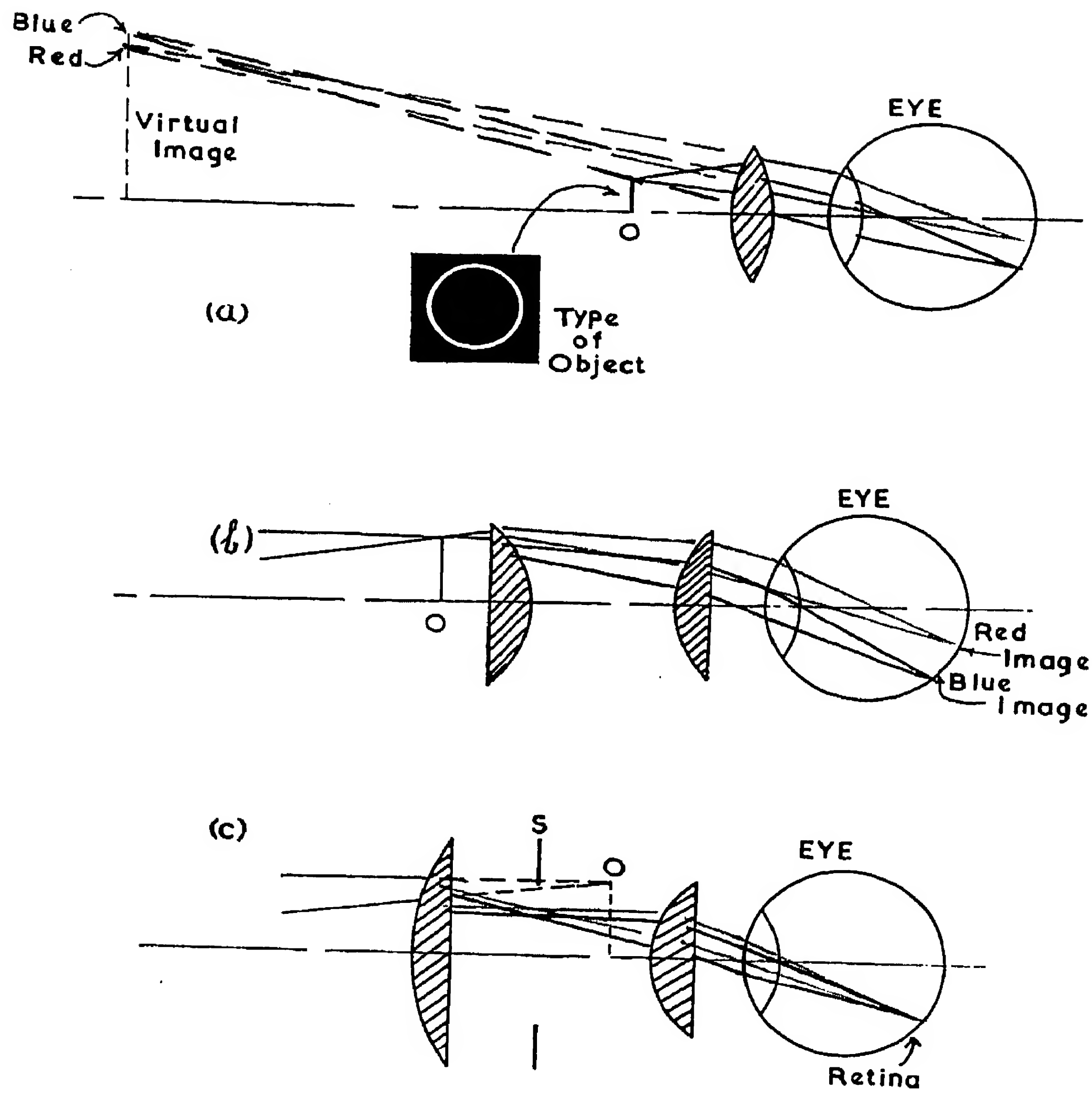


Fig. 18.

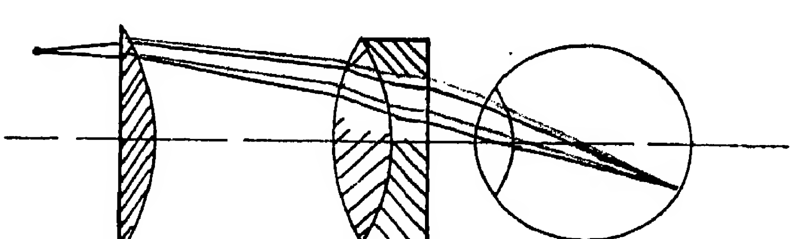


Fig. 23.

formed and where all the light admitted by the objective is concentrated within the smallest possible circle. The pupil of the eye must be placed very close to this point if the *whole* field is to be seen clearly, and this is only possible if the eye-point is far enough from the eye lens to allow room for the eyelids and eyelashes. Five or six millimetres is about the minimum, whilst eight millimetres is about the most comfortable distance; in certain low-power eyepieces where the eye-point may be further away than this, an eye-cap is fitted to the eyepiece tube in order to locate the exit-pupil more easily by "feeling" this cap with the eyelashes.

The chief advantage of the Huygenian eyepiece is that it can be completely freed from chromatic difference of magnification or transverse chromatic aberration (as it is sometimes called) and in consequence is free from false one-sided colour in the outer parts of the field. The removal of this aberration is of considerable importance when dealing with instruments which are used purely for observational work with "white" light (e.g., telescopes and microscopes); hence Huygenian eyepieces are greatly to be preferred for use with such instruments.

The Ramsden type, however, can be better corrected for all the other aberrations and is therefore suitable for use with instruments in which monochromatic light is employed, such as spectroscopes, etc. On this account also, it is more suitable when micrometric devices are to be used in the focal plane of the eyepiece, for less distortion of the image occurs with this eyepiece than with the Huygenian. But the serious amount of transverse chromatic aberration inherent in the Ramsden eyepiece makes it unpleasant to use for observational work, except when this type is achromatized (see later).

In order to understand more clearly what is meant by chromatic difference of magnification, let us refer to Fig. 18a, b and c. Imagine that at O (Fig. 18a) there is an object consisting of a transparent ring in an opaque background illuminated from behind, and that this object is viewed with a short-focus lens of one glass only; then the eye will see a virtual image of this ring as indicated in the diagram but owing to the dispersion of the light in passing through the single lens the image of the ring will appear bordered with an outer blue fringe of colour and an inner red fringe. Also from the diagram it will be apparent that the height of the red image from the axis will be less than that of the blue, which effect is clearly one of chromatic difference of magnification.

If the same kind of object is placed in front of a Ramsden eyepiece (Fig. 18b) the light will be dispersed by the two lenses and hence will give a greater angular deviation between the red and blue rays as they leave the eye lens to enter the eye. Thus the red and blue virtual images will be further separated, resulting in a greater amount of chromatic difference of magnification than with the ordinary single lens hand-magnifier.

In Fig. 18c it will be assumed that an objective is forming an image of the ring object in the plane O, and by interposing the field lens of an Huygenian eyepiece the image will now be brought into the plane of the diaphragm S but it will undergo dispersion in doing so and give a red image which is larger in diameter than the blue. As the red and blue rays pass on they can be made (by the design) to emerge from the eye lens parallel to one another, and are therefore focused on the retina of the eye as a non-coloured image and consequently there is no longer present the defect known as chromatic difference of magnification or transverse chromatic aberration.

As this defect may therefore be more easily corrected with the Huygenian eyepiece we will at first confine our attention to the design of this type. The other aberration which has to be eliminated (or kept to within very small limits) in eyepiece design is coma.

The sequence of operations in the procedure of the design will be as follows :—

(1) Determination of the focal lengths of the eye lens and field lens from the stipulated equivalent focal length of the eyepiece system.

(2) Alteration in the ratio of the focal lengths of field lens and eye lens.

Experience shows that f_b'/f_a' may be taken as 2 for a usual range of magnification, such as from about 15 to 38 times. It may, however, vary from as high as 2·3 to as low as 1·4.

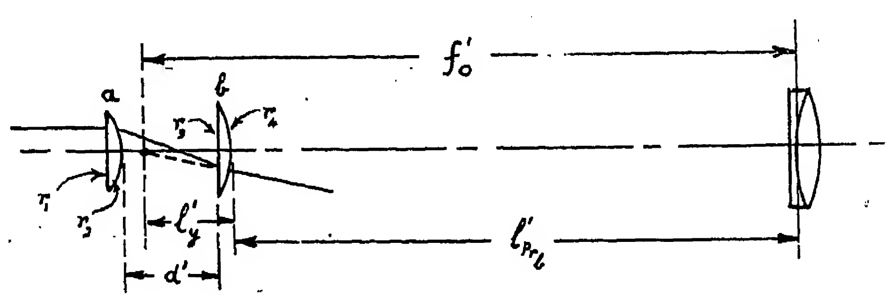


Fig. 19.

(3) Alteration in the separation of the field lens and eye lens as indicated by Siedel Aberration VI (See Conrady's *Applied Optics*, pp. 303 to 363).

(4) Determination of the chromatic aberration, spherical aberration and Offence against the Sine Condition, for three separations of the lenses by ray-tracing methods. Also the equivalent focal length (E.F.L.) in each case.

(5) Determining for each of these separations the value of the tube-length $l'pr_b$ (see Fig. 19) which by the sine condition gives zero coma.

(6) Determination of the transverse chromatic aberration for each separation of the eyepiece lenses.

(7) Plotting all these aberrations on one graph, and choosing the best solution as dictated by the permissible limits on the transverse chromatic aberration.

Huygenian Eyepieces

We will now illustrate numerically the various steps in designing our Huygenian eyepiece by the following example. Let us ask that the E.F.L. of the eyepiece shall be $2 \cdot 54$ cm. (i.e., a $\times 10$) and that we are going to use a fairly representative crown glass for both field lens and eye lens, with optical constants :—

N_D	$N_F - N_C$	V	$N_D - N_C$	$N_F - N_D$
1·5153	0·0090	57·2	0·00254	0·00646

The thin lens formula for the E.F.L. of the eyepiece $= \dfrac{f'_a \cdot f'_b}{f'_a + f'_b - d}$

where f'_a and f'_b are the focal lengths of the eye lens and field lens respectively and d the separation of the lenses.

Assuming an initial ratio of f'_b to f'_a as 2 : 1, and a separation of the lenses as $\frac{3}{2} f'_a$ we can obtain the focal lengths f'_a and f'_b by substitution in the above formula as follows :—

$$2 \cdot 54 = \frac{f'_a \times 2 f'_a}{f'_a + 2 f'_a - \frac{3}{2} f'_a} = \frac{2(f'_a)^2}{\frac{3}{2} f'_a} = \frac{4}{3} f'_a.$$

Hence, $f'_a = 1 \cdot 90$ cm. and therefore $f'_b = 3 \cdot 80$ cm.

From $\dfrac{1}{f} = (N-1)\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$ we can determine the radii r_2 and r_4 of the curved surfaces of the lenses (assuming they are plano-convex), namely for the eye lens $\dfrac{1}{1 \cdot 90} = (1 \cdot 5153 - 1)\left(\dfrac{1}{\infty} - \dfrac{1}{r_2}\right)$ so that $r_2 = -0 \cdot 979$ cm. and for the field lens $r_4 = -1 \cdot 958$ cm.

It will be sufficiently close to call these radii $1 \cdot 00$ cm. and $2 \cdot 00$ cm. respectively, as this will only make a difference of $0 \cdot 04$ cm. in the E.F.L. of the eyepiece ; moreover, it is much more likely that there would be grinding and polishing tools in the glass workshop of these radii than of the odd sizes such as $0 \cdot 979$ and $1 \cdot 958$ cm.

We now determine the most suitable separation d' of the lenses by applying the equation relating to Seidel Aberration VI from

$$d' = \frac{f'_a + f'_b \times \dfrac{V_b}{V_a}}{1 + \dfrac{V_b}{V_a} + \left(\dfrac{f'_a}{l_a} - \dfrac{f'_b \cdot V_b}{l'_{pr_b} \cdot V_a}\right)}$$

but as l_a is ∞ (i.e., parallel light emerging from the eye lens to the eye) and as the two lenses are of the same glass (i.e., $V_b = V_a$) the equation reduces itself to :—

$$d' = \frac{(f'_a + f'_b)}{\left(2 - \dfrac{f'_b}{l'_{pr_b}}\right)}$$

This immediately introduces a problem, because l'_{pr_b} (the tube-length) see Fig. 19, is to be determined by the zero-coma condition and is therefore not yet known. Its influence on d', however, is not very large and as we are at the moment assuming the eyepiece is to be used with a telescope objective, we may take the average focal length of the latter for a small astronomical telescope as about three feet or say 95 cm. and use this as l'_{pr_b}.*

Thus d' now becomes $= \dfrac{(1 \cdot 90 + 3 \cdot 80)}{2 - \left(\dfrac{3 \cdot 80}{95}\right)} = 2 \cdot 90$ cm.

We can now begin to prepare the prescription for carrying out the trigonometrical ray-tracing in order to obtain the axial chromatic and spherical aberrations, and in order to be able to apply the sine condition. It is necessary to do this for three different separations of the lenses and we will choose separations on each side of the $2 \cdot 90$ cm. (as determined for d' above) namely at $2 \cdot 60$ cm. and $3 \cdot 20$ cm.

The specification will be, therefore (see Fig. 20) :—

$r_1 = \infty$

$\leftarrow$ Axial thickness $= 0 \cdot 15$ cm.

$r_2 = -1 \cdot 00$ cm.

Air space $d'_2 = \begin{cases} 2 \cdot 60 \text{ cm.} \\ 2 \cdot 90 \text{ cm.} \\ 3 \cdot 20 \text{ cm.} \end{cases}$

$r_3 = \infty$

$\leftarrow$ Axial thickness $= 0 \cdot 30$ cm.

$r_4 = -2 \cdot 00$ cm.

$Y = 0 \cdot 20$ cm.

* N.B.—For microscope objectives $l'pr_b$ would more nearly be 16 to 18 cm.

The glass as given on page 57.

In eyepiece design, a certain amount of time may be saved in the ray-tracing work (without incurring any loss of accuracy) by employing a so-called three-ray method. This involves tracing two rays at $0 \cdot 8$ of the semi-aperture instead of at the full aperture, one of which is in yellow-green light (N_y) and the other in blue light (N_v); and one paraxial ray in N_y. By this means we can obtain a measure of the chromatic aberration as $(L'_y - L'_v)$ and of the spherical aberration as $(l'_y - L'_y)$.

In order to obtain the required refractive index values

$$N_y = N_D + 0 \cdot 188\,(N_F - N_C) = 1 \cdot 5153 + 0 \cdot 0017 = 1 \cdot 5170.$$

and $$N_v = N_y + (N_F - N_C) = 1 \cdot 5170 + 0 \cdot 0090 = 1 \cdot 5260.$$

Taking the semi-aperture of the eye-lens as $0 \cdot 25$ cm., the value of Y for the three-ray method will be $0 \cdot 80 \times 0 \cdot 25 = 0 \cdot 20$ cm.

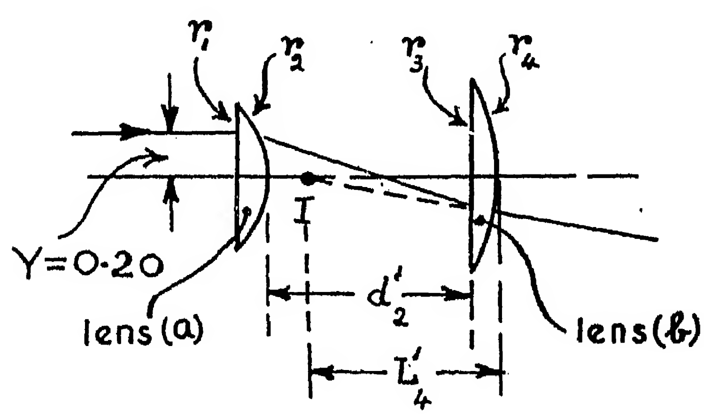


Fig. 20.

Referring to Fig. 20 and to Calculation No. 16 it will be seen that the incident parallel ray at $Y = 0 \cdot 20$ cm. will meet the flat surface ($r_1 = \infty$) normally and will therefore pass on undeviated so that the ray-tracing really begins at surface No. 2 with $r_2 = -1 \cdot 000$ cm. After passing through this surface the air-space d' must be subtracted from L'_2 in order to obtain the new L_3 and the ray-tracing continued through the field lens, giving the final L'_4 which will be found to have a negative sign. This indicates a virtual image as shown at the point I (Fig. 20) and thus the values for L'_y, l'_y and L'_v will all be negative, representing for a lens-separation of $2 \cdot 60$ cm. an axial chromatic aberration $(L'_y - L'_v)$ of $+ 0 \cdot 0645$ cm. and spherical aberration $(l'_y - L'_y)$ of $+ 0 \cdot 1593$ cm.

The optical tolerances when using the three-ray method of tracing are derived in a later paragraph.

Another separation of the lenses, namely $2 \cdot 90$ cm., must now be taken and ray-tracing carried out as before; but it will be seen that a considerable amount of the work has already been done, for the tracing through the eyelens

remains the same (care being taken in this case to subtract 2·90 cm. from the L'_2 values before proceeding through surface r_3) so that it is only necessary to trace through the two surfaces of the field lens in turn for each separation of 2·90 cm. and 3·20 cm.

The results so far will be found to be as follows (Table III) :—

TABLE III

Separation of lenses d' =		2·60 cm.	2·90 cm.	3·20 cm.	Optical Tolerances
Intersection lengths L'_y	=	$-1{\cdot}271$	$-1{\cdot}864$	$-2{\cdot}611$	
Intersection lengths l'_y	=	$-1{\cdot}112$	$-1{\cdot}664$	$-2{\cdot}354$	
Intersection lengths L'_v	=	$-1{\cdot}336$	$-1{\cdot}951$	$-2{\cdot}734$	
Axial Chrom. Aberration $(L'_y - L'_v)$	=	$+0{\cdot}065$	$0{\cdot}087$	$0{\cdot}123$	$\pm 0{\cdot}0068$ cm.
Axial Sph. Aberration $(l'_y - L')$	=	$+0{\cdot}159$	$0{\cdot}200$	$0{\cdot}257$	$\pm 0{\cdot}0174$ cm.
E.F.L. of eyepiece system	=	2·49	2·77	3·11	

Having reached the stage given by Table III, we now apply Sine Theorem II to determine the " tube-length " $l'pr_b$ at which the eyepiece will be free from coma. Sine Theorem II equation is :—

$$l'pr_k = l'_k - \frac{(l'_k - L'_k)(1 - OSC')}{(1 - OSC') - \left(\frac{\sin U_1}{u_1} \cdot \frac{u'_k}{\sin U'_k}\right)}$$

The above equation may be changed into a more convenient form for use in eyepiece design, namely

$$l'pr_k - l'_k = f'_o = \frac{(l'_y - L'_y)(1 - OSC')}{\dfrac{u'_y}{\sin U'_y} - (1 - OSC')} \quad . \quad . \quad . \quad \text{E.P.(1)}$$

At first it would seem desirable to solve for OSC' = zero in this sine theorem equation, but this would give complete coma correction in the central part of the field and undesirable amounts of coma would appear in the outer parts of the field. Therefore a compromise has to be made, and experience has led to the conclusion that it is best to solve for $OSC' = 0{\cdot}2 \sin^2 U'_y$ giving zero coma at about twelve degrees from the centre of the field.

We will, therefore, calculate the values for $OSC' = 0{\cdot}2 \sin^2 U'_y$ for the three separations of eye lens and field lens. (Table IV.)

These values for OSC' are now put into the equation E.P.(1) in order to determine the focal length f_0' of the objective which will be most suitable for each separation of the eyepiece lenses. This gives :—

Separations	=	2·60	2·90	3·20 cm.
Focal length f_0'	=	93·59	38·46	26·60 cm.

TABLE IV

Separations	=	2·60	2·90	3·20
log final sin U_y'	=	8·90464*	8·85734*	8·80427*
log 0·2 + 2 log sin U_y'	= =	9·30103 7·80928	9·30103 7·71468	9·30103 7·60854
log OSC'	=	7·11031	7·01571	6·90957
OSC'	=	0·00129	0·00104	0·00081

*From ray-tracings.

Having determined the three focal lengths of the objectives, we then use these values in Sine Theorem V in order to obtain the state of correction of the eyepieces with reference to transverse chromatic aberration (g_7).

The modified form of Sine Theorem V for eyepiece design is as follows :—

$$g_7 = \left(1 - \frac{\sin U_y'}{\sin U_v'}\right) + \left[\frac{\sin U_y'}{\sin U_v'} \cdot \frac{(L_y' - L_v')}{f_0' + (L_y' - L_v')}\right]$$

Substituting the numerical values from the ray-tracings in the above, we find :—

Separations	=	2·60	2·90	3·20 cm.
Trans. Chrom. Ab. (g_7)	=	+ 0·00124	− 0·00112	− 0·00392

Collecting all the various results we have obtained, we can now proceed to plot these all on one graph (see Fig. 21) using the separation of the eyepiece lenses as abscissa and the other corresponding values as ordinates. We may include also one further set of values, namely the magnifications in each case from the ratio of focal length of objective to focal length of eyepiece $\left(\frac{f_0'}{f'_{E.P.}}\right)$ which will be found to be $37·58\times$, $13·90\times$ and $8·55\times$ respectively for the three previously mentioned separations. For the sake of plotting, however, it will be found more convenient to take the reciprocal of these values $\frac{1}{M}$, namely 0·0266, 0·0719 and 0·1170.

With the graphs now drawn as in Fig. 21, the question arises as to how the most suitable choice of a solution should be made. The transverse chromatic

aberration (or chromatic difference of magnification) is the dominating factor as mentioned earlier ; but instead of choosing a zero value for this, it is found desirable to choose a solution with a little *positive* transverse chromatic aberration at the centre of the field in order to compensate the negative higher aberration at the outer parts of the field. Experience advises a best g_7 value of $+0 \cdot 001$, so that marking this point A on the g_7 graph we may draw a straight line through this point parallel to the ordinates passing through all the other curves. Reading off from the latter the corresponding values we find are :—

(1) the best separation of the lenses $= 2 \cdot 63$ cm.

(2) the focal length of the eyepiece $f'_{E.P.} = 2 \cdot 52$ cm.

(3) the chromatic aberration $= + 0 \cdot 066$ cm.

(4) the spherical aberration $= + 0 \cdot 162$ cm.

(5) $\dfrac{1}{M} = 0 \cdot 031$ and therefore $M = 32 \cdot 3 \times$

and if we add to these the radii of the curved surfaces of the plano-convex eye lens and field lens, namely $r_2 = 1 \cdot 00$ cm. and $r_4 = 2 \cdot 00$ cm. respectively, we get the complete specification of the eyepiece thus designed.

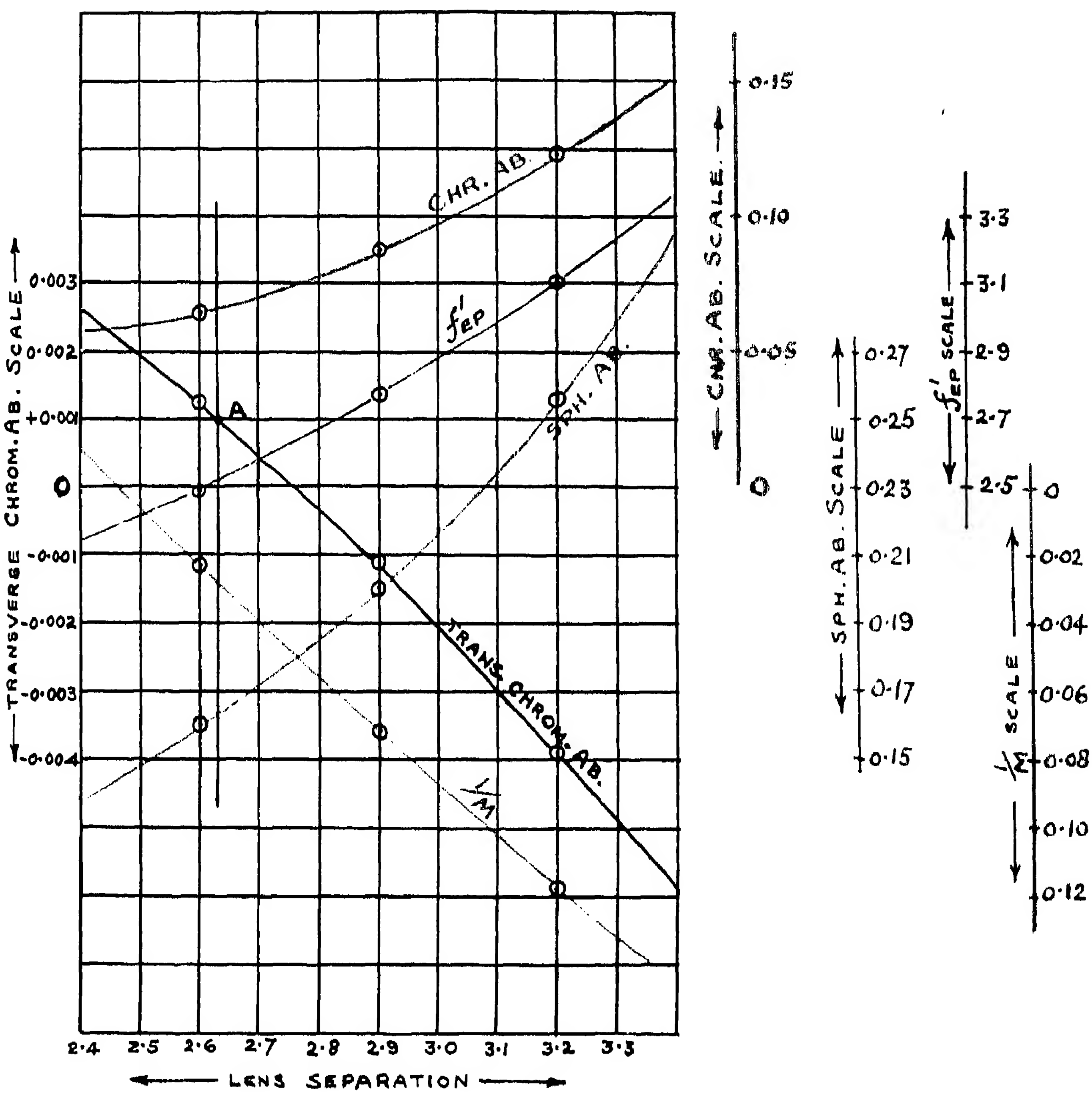
It will be seen that both the chromatic aberration and the spherical aberration given by the eyepiece is considerably greater (about ten times) than the permissible tolerances, and if such an eyepiece were used with a chromatically and spherically corrected objective the complete instrument would be gravely imperfect. Thus, our objective would have to be designed to give the prescribed *over*-correction in order to compensate for the *under*-correction of the eyepiece. The importance of designing and using the objective and eyepiece in conjunction with one another is therefore clearly brought out.

N.B.—It should be pointed out that, in accordance with the treatment of "transverse chromatic aberration" (see p. 491 and sections (64) and (65) of Conrady's *Applied Optics*) this defect will be invisible if we allow a tolerance of $\pm 0 \cdot 001$ from the best value, namely $+ 0 \cdot 001$ This gives the limits as $+ 0 \cdot 002$ to zero ; consequently these limits will decide the range of magnifications with which the ratio of field lens to eye lens of 2 : 1 may be safely used. Thus from the graph, the range is $\dfrac{1}{M} = 0 \cdot 014$ to $0 \cdot 050$ or $M = 74$ to $M = 20$.

Optical Tolerances when applied to the Three-ray Method

From the optical tolerances given on page 32 it will be seen that when the extreme marginal ray is traced, the 'focal range' is equal to $\dfrac{1 \lambda}{N' \, . \sin^2 U'_M}$.

But as the focal range diminishes inversely as the square of the aperture, we must multiply its value found for the U' of the $0 \cdot 8$ ray by $(0 \cdot 8)^2$ or $0 \cdot 64$.



CHR. AB.
f'_{EP}
SPH. AB.
A
TRANS. CHROM. AB.
1/M
TRANSVERSE CHROM. AB. SCALE
0·003
0·002
+0·001
0
-0·001
-0·002
-0·003
-0·004
2·4
2·5
2·6
2·7
2·8
2·9
3·0
3·1
3·2
3·3
LENS SEPARATION
CHR. AB. SCALE
0·15
0·10
0·05
0
SPH. AB. SCALE
0·27
0·25
0·23
0·21
0·19
0·17
0·15
f'_{EP} SCALE
3·3
3·1
2·9
2·7
2·5
1/M SCALE
0
0·02
0·04
0·06
0·08
0·10
0·12

Fig. 21.

Hence, for the 0·8 aperture three-ray method, the Chromatic Aberration Tolerance $= \pm \dfrac{0 \cdot 64\lambda}{N' . \sin^2 U'_{0\cdot 8}}$. For calculations at full aperture the spherical aberration tolerance is $\dfrac{4\lambda}{N' . \sin^2 U'_M}$. In the three-ray method we calculate for the 0·8 aperture ray, and the $\sin^2 . U'$ of this will be only 0·64 of the marginal $\sin^2 U'_M$. At the same time the spherical aberration at 0·8 of the aperture is only 0·64 of the marginal value. Combining the two corrections we have $(0 \cdot 64)^2 = 0 \cdot 41$.

Therefore the tolerance worked out with the sin U' of the 0·8 ray must be put at :—

$$\text{Spherical Aberration Tolerance} = \pm \frac{1 \cdot 64\lambda}{N' . \sin^2 U'_{0\cdot 8}}.$$

Applying these to our calculations for lens-separation of 2·90 we get :—

$$\text{Chrom. Ab. Tol.} = \frac{0 \cdot 64 \times 0 \cdot 000055}{1 \times (\sin^2 4^\circ\text{–}7'\text{–}44'')} = \pm\, 0 \cdot 0068 \text{ cm.}$$

$$\text{and Sph. Ab. Tol.} = \frac{1 \cdot 64 \times 0 \cdot 000055}{1 \times \sin^2 (4^\circ\text{–}7'\text{–}44'')} = \pm\, 0 \cdot 0174 \text{ cm.}$$

Ramsden Eyepieces

In the ordinary Ramsden eyepiece of two plano-convex lenses (see Fig. 17b) and made from the same kind of glass, the eye lens obviously must have a focal length which is larger than the air space in order to secure the external focal plane, or positive l'_b, which is the chief advantage and the reason for the existence of this eyepiece. On the other hand it can be shown that the field lens practically always has to have a longer focal length than the eye lens in order to secure freedom from coma and flatness of field.

Now the condition of achromatism of magnification (from Seidel Aberration VI) demands an air space between the lenses which is necessarily larger than f'_a: this contradicts the demand for an accessible or *real* focal plane, and as the latter demand cannot be evaded it follows that ordinary Ramsden eyepieces cannot be freed from transverse chromatic aberration.

Naturally we shall attempt to reduce this unavoidable defect to the utmost possible extent by fixing the air space at the largest possible fraction of f'_a. Experience shows that a safe value for the separation of the lenses, d', may be taken as $0 \cdot 7\, f'_a$: lower values down to $\frac{1}{2} f'_a$ are occasionally met with, but they only lead to an aggravation of the transverse chromatic aberration.

The air space being thus fixed, there is only one liberty left for varying the correction of a Ramsden eyepiece, namely, the ratio of the focal lengths

of eye lens and field lens; consequently only one aberrational condition can be satisfied by the choice of the appropriate pupil position, and as this we choose freedom from coma because it carries with it the greatest possible flattening of the field.

We will now proceed with the design of an ordinary Ramsden eyepiece which can be carried out by the methods and formulæ used in the preceding section, but it becomes much simpler because there is no variation in air space separation between the lenses.

As our example let us take a Ramsden eyepiece with the same equivalent focal length as used for the Huygenian, namely $f'_{EP} = 2 \cdot 50$ cm. The ratio of the focal length of field lens to eye lens in Ramsden eyepieces is frequently taken as 1 : 1, but slightly better results are obtained by using a ratio of 1·25 : 1. Choosing this latter ratio, the focal length of each lens is first determined from the thin lens formula :—

$$\text{E.F.L.} = \frac{f'_a \cdot f'_b}{f'_a + f'_b - d} \text{ so that}$$

$$2 \cdot 50 = \frac{f'_a \cdot 1 \cdot 25 f'_a}{f'_a + 1 \cdot 25 f'_a - 0 \cdot 7 f'_a} = \frac{1 \cdot 25 (f'_a)^2}{1 \cdot 55 f'_a} = 0 \cdot 806 f'_a$$

Therefore $f'_a = 3 \cdot 10$ cm.

and $f'_b = 1 \cdot 25 \times 3 \cdot 10 = 3 \cdot 88$ cm.

From these focal lengths the radii of the plano-convex lenses will be found from $\frac{1}{f} = (N-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ to be $r_2 = 1 \cdot 60$ cm. and $r_4 = 2 \cdot 00$ cm.

And the air space will be $0 \cdot 70 \times 3 \cdot 10 = 2 \cdot 17$ cm.

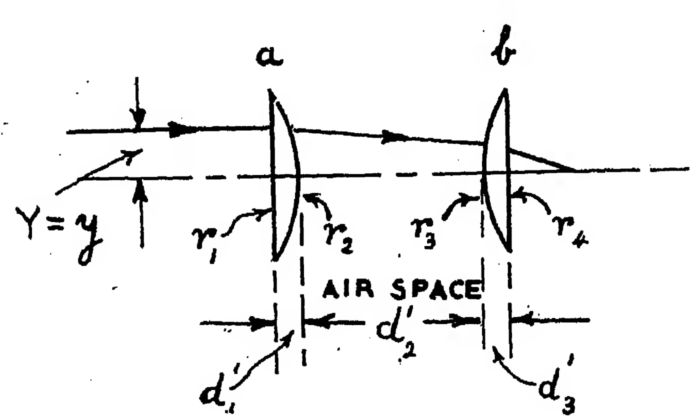


Fig. 22.

The three-ray method for ray tracing will again be used here to determine the axial chromatic and spherical aberrations and to enable

the sine condition to be applied. The specification for this will be, therefore :—

$r_1 = \infty$

$\leftarrow$ Axial thickness $d'_1 = 0 \cdot 15$ cm.

$r_2 = -1 \cdot 60$ cm.

See Fig. 22. Air space $d'_2 = 2 \cdot 17$ cm.

$r_3 = +2 \cdot 00$ cm.

$\leftarrow$ Axial thickness $d'_3 = 0 \cdot 20$ cm.

$r_4 = \infty$

Glass : as used for the Huygenian, $N_y = 1 \cdot 5170$

$N_v = 1 \cdot 5260$

$Y = y = 0 \cdot 20$ cm.

The result of the ray-tracing gives the following values :—

$L'_y = 0 \cdot 5767$ cm. ; $l'_y = 0 \cdot 6145$ cm. ; $L'_v = 0 \cdot 5400$ cm.

$U'_y = 4°–35'–44''$; $u'_y = 0 \cdot 080076$; $U'_v = 4°–38' – 16''$

Chromatic Aberration $(L'_y - L'_v) = +0 \cdot 0367$ cm.

Spherical Aberration $(l'_y - L'_y) = +0 \cdot 0378$ cm.

Equivalent Focal Length (f'_{EP}) of complete eyepiece $= 2 \cdot 49(8)$ cm.

As the focal length of our Ramsden eyepiece and the initial Y value employed are almost exactly similar to the Huygenian, the F/ratio of the two eyepieces is similar and therefore a direct comparison can be made between the performances given by each of them. It will be noted from the foregoing results that the spherical aberration of the Ramsden is only about one fifth of that given by the Huygenian, and there is also a definite but smaller advantage in the longitudinal chromatic aberration.

Proceeding with the numerical work we now use equation EP(1) in order to determine the focal length of the objective f'_0 most suitable for the chosen separation of the eyepiece lenses. This involves the OSC' value $= 0 \cdot 2 \sin^2 U'_y$, which utilizing the U'_y from the ray-tracing gives $OSC' = +0 \cdot 00128$; and hence f'_0 will be found to be $53 \cdot 9$ cm. Variation of the ratio f'_a to f'_b would allow of other values of f'_0.

From the modified form of Sine Theorem V (as given on page 61) the chromatic difference of magnification is then obtained from

$$g_7 = \left(1 - \frac{\sin U'_y}{\sin U'_v}\right) + \left[\frac{\sin U'_y}{\sin U'_v} \cdot \frac{L'_y - L'_v}{f'_0 + (L'_y - L'_v)}\right]$$

which gives a value of $+0 \cdot 0098$. This value is distinctly high when compared with that given by the Huygenian and bears out the remarks at the opening of this chapter, namely that the most serious defect of the ordinary Ramsden eyepiece is its large amount of transverse chromatic aberration.

Nothing is to be gained by making the two lenses of different glasses; as an experiment one may try this by using (say) a crown glass for the eye lens and a flint glass for the field lens of the following optical constants:

	N_D	$N_F - N_C$	V
Eye lens	1·5153	0·0090	57·2
Field lens	1·7167	0·0243	29·5

and by repeating the calculations already described, one finds for such an eyepiece of the same focal length that:—

$L'_y = 0\cdot5917$ $\quad l'_y = 0\cdot6296$ $\quad L'_v = 0\cdot5542$
$U'_y = 4°\text{–}35' - 54''$ $\quad u'_y = 0\cdot080127$ $\quad U'_v = 4°\text{–}39'\text{–}16''$

and thus the axial chromatic aberration is $+0\cdot0375$ cm., the axial spherical aberration is $+0\cdot0379$ cm., the $f'_{O.G.} = 47\cdot3$ cm., and the chromatic difference of magnification is $+0\cdot0128$. As would be expected, the higher dispersion of the field lens merely aggravates this latter aberration and makes it greater than with the ordinary Ramsden eyepiece using both lenses of the same glass.

If, however, one uses a material of less dispersive power than even crown glass for the lenses, both the axial and transverse chromatic aberration may be reduced. Materials such as methyl methacrylate (Perspex) or lithium fluoride might be tried in this connection; if the latter substance is used for a Ramsden eyepiece of similar power, the following values will be found from the ray-tracing:—

$L'_y = 0\cdot5186$; $\quad l'_y = 0\cdot5693$; $\quad L'_v = 0\cdot5000$
$U'_y = 4°\text{–}48'\text{–}13''$; $\quad u'_y = 0\cdot083840$; $\quad U'_v = 4°\text{–}49'\text{–}26''$

Thus, the axial chromatic aberration $= +0\cdot0186$ cm., the axial spherical aberration $= +0\cdot0507$ cm., and the transverse chromatic aberration is found to be $+0\cdot0051$. This value is about half that obtained with the Ramsden utilizing crown glass for the two lenses, but is still rather too much outside the permissible tolerance for this aberration (namely $+0\cdot0020$) but an interesting comparison is afforded by studying the results given by three such Ramsden eyepieces.

Achromatized Ramsden Eyepiece

The outstanding defect of the ordinary Ramsden eyepiece (namely, the high value of its transverse chromatic aberration) can be reduced to a relatively harmless amount by replacing the simple plano-convex eye lens by an achromatic cemented combination. By referring to Fig. 23 and comparing this with Fig. 18(b) it will be seen that the emerging coloured rays can be rendered parallel and will therefore be brought to a single focus on the retina of the eye as in the case of the Huygenian illustrated in Fig. 18(c); thus, the various advantages of low aberrational values obtained with the "ordinary" Ramsden

eyepiece may be maintained whilst at the same time the chromatic difference of magnification can be considerably reduced.

From the diagram of the achromatic eye lens it can be readily visualized and from the ray-tracing it can be proved, that the rays make extremely large angles of incidence at the contact surface (angles of 35° are not uncommon) and such a state of affairs tends to produce large amounts of higher aberration. These can be kept to within a manageable magnitude by making the difference in refractive indices $N' - N$ at the contact surface as small as possible, and to this end it is desirable to use glasses such as a medium barium crown combined with a rather light flint. A suitable choice of these glasses for the eye lens is the chief secret of success in the design of this form of eyepiece.

It may be necessary in certain circumstances, such as with high-power eyepieces of wide angular field, to employ a triple component eye lens in order to keep down the serious effect of the higher aberrations.

Taking, therefore, as a numerical example of this form of eyepiece, one of similar power to those we have already dealt with (namely $f'_{EP} = 2\cdot50$ cm.) we shall have as before that the focal length of the eye lens is to be $3\cdot10$ cm., for the field lens $3\cdot88$ cm. and a separating air space of $2\cdot17$ cm.

Referring to Fig. 25 we might start therefore with a right-to-left ray tracing through the field lens using an object distance $L = 0\cdot540$ cm. and initial $U = 4°-38'-16''$ (chosen quite arbitrarily from the previous calculation on the Ramsden eyepiece) in order to find the amount of the chromatic aberration introduced by the single glass field lens, but as this quantity is so small, namely $0\cdot002$ cm., it is quite unnecessary to worry about this slight amount of undercorrected chromatic aberration at the outset and simply go on to find the total curvature of the two components of the eye lens for the desired focal length of $3\cdot10$ cm. and then adjust the last radius (if necessary) to give complete achromatism of the whole system. The two glasses we will use for the present example will be :—

	N_D	$N_F - N_C$	V
Light flint (lens a. See Fig. 25) ..	1·6041	0·01599	37·8
Medium barium crown (lens b. See Fig. 25)	1·5744	0·00995	57·7
Hard crown for the field lens ..	1·5153	0·0090	57·2

Calculating the total curvature of lens a and b of the eye lens, we have

$$R_a = \frac{1}{f'.(V_a - V_b)\delta N_a} = \frac{1}{3\cdot10 \times -19\cdot9 \times 0\cdot01599} = -1\cdot014$$

$$R_b = \frac{1}{f'.(V_b - V_a)\delta N_b} = \frac{1}{3\cdot10 \times 19\cdot9 \times 0\cdot00995} = +1\cdot629$$

Assuming a flat first surface of the flint lens, then $r_1 = \infty$; and from $R_a = R_1 - R_2$, $r_2 = +0 \cdot 986$ cm.; and as the lens is a cemented doublet $r_3 = +0 \cdot 986$ cm. and r_4 will be found as $-1 \cdot 626$ cm.

From a scale drawing the axial thicknesses will be $d_1' = 0 \cdot 15$ cm. and $d_2' = 0 \cdot 25$ cm. With $Y = 0 \cdot 20$ as before, the specification is ready for ray-tracing. The three-ray method may be employed, the required refractive indices being $N_y = 1 \cdot 6071$, $N_v = 1 \cdot 6231$ for the flint lens, and $N_y = 1 \cdot 5762$, $N_v = 1 \cdot 5861$ for the crown lens. The result of the calculations after the three rays have passed through this achromatic lens are as follows :—

$L_y' = +0 \cdot 873$ cm.; $l_y' = +0 \cdot 924$ cm.; $L_v' = +0 \cdot 874$ cm.
$U_y' = 3°\text{–}45'\text{–}40''$; $u_y' = 0 \cdot 064957$; $U_v' = 3°\text{–}45'\text{–}48''$

Applying the air-space thickness $d_4' = 2 \cdot 17$ cm., the tracing is continued on through the field lens whose specification will be $r_5 = +2 \cdot 000$ cm., $r_6 = \infty$, $d_5' = 0 \cdot 20$ cm., with refractive indices $N_y = 1 \cdot 5170$ and $N_v = 1 \cdot 5260$.

The final result of the ray tracing will be found to be :—

$L_y' = +0 \cdot 5797$; $l_y' = 0 \cdot 6144$; $L_v' = 0 \cdot 5789$;
$U_y' = 4°\text{–}36'\text{–}46''$; $u_y' = 0 \cdot 080473$; $U_v' = 4°\text{–}37'\text{–}53''$

giving an axial chromatic aberration $(L_y' - L_v')$ of $+0 \cdot 0008$ cm., and spherical aberration $(l_y' - L_y')$ of $+0 \cdot 0347$ cm.

It will be seen that the complete eyepiece system is well achromatized; for the tolerance (3-ray method) $= \dfrac{0 \cdot 64\ \lambda}{N' \,.\, \sin^2 U'_{0 \cdot 8}} = 0 \cdot 0054$ cm., whereas the actual chromatic aberration is only $0 \cdot 0008$ cm. The equivalent focal length is $2 \cdot 48(5)$ cm.

The determination of the focal length of the objective most suitable for the reduction of coma with this eyepiece is obtained from :—

$$f'_{OG} = \frac{(l_y' - L_y')(1 - OSC')}{\dfrac{u_y'}{\sin U_y'} - (1 - OSC')}, \text{ and taking } OSC' = 0 \cdot 2 \ \sin^2 U_y'$$

$$f'_{OG} = \frac{0 \cdot 0347 \times 0 \cdot 9987}{1 \cdot 0006 - 0 \cdot 9987} = 18 \cdot 24 \text{ cm.}$$

This gives a magnification ratio $\dfrac{f'_{OG}}{f'_{EP}}$ of $7 \cdot 9$ times which is close to the usual magnification employed on binocular telescopes for which the eyepiece in this example is intended.

The chromatic difference of magnification is obtained from

$$g_\gamma = \left(1 - \frac{\sin U_y'}{\sin U_v'}\right) + \left[\frac{\sin U_y'}{\sin U_v'} \cdot \frac{L_y' - L_v'}{f_o' + (L_y' - L_v')}\right]$$

which gives a value of $+0 \cdot 0039$. It will be noticed, therefore, how this aberration has been reduced when compared with the value (namely $0 \cdot 0098$) obtained with the ordinary non-achromatized Ramsden (see page 65).

It should be pointed out that the chromatic difference of magnification may be still further reduced if the eyepiece is slightly over-corrected for axial achromatism; this makes a much smaller difference in the final angles U'_y and U'_v and thus a value more nearly unity for $\dfrac{\sin U'_y}{\sin U'_v}$ which is the controlling factor in the g_7 equation above. This should certainly be tried by any who wish to carry out a design of eyepiece other than the illustrative example given here.

It will be noted that in the case of all these eyepieces dealt with in this section that both the "tube-length" l'_{pr} most suitable to the elimination of coma, and the transverse chromatic aberration, have been obtained by their respective analytical formulæ. Whilst this is perfectly reliable for the simpler types of eyepiece such as the ordinary Huygenian and Ramsden, it is advisable in the case of the more complex types such as the achromatized Ramsden and those eyepieces of higher power and wider field of view, to test the exact amount of the coma and chromatic difference of magnification by trigonometrical means. We will therefore do this with the achromatized Ramsden we have under consideration here.

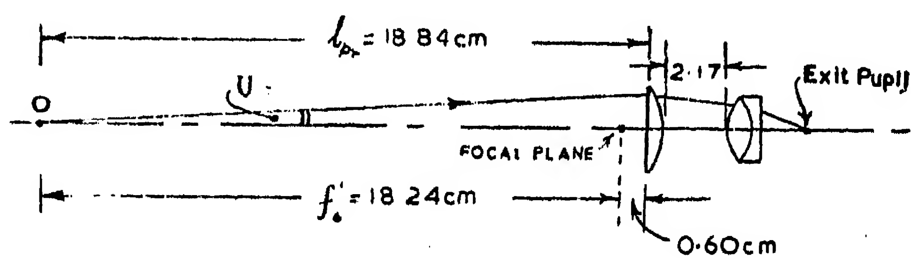


Fig. 24.

The first step involves finding the position of the exit pupil, from which parallel rays may be traced at a given obliquity corresponding to the angular field concerned. Referring to Fig. 24 a ray is traced from O at a distance l'_{pr} equal to f'_0 plus the distance of the focal plane from the flat first surface of the field lens (namely $0 \cdot 60$ cm.); this gives $l'_{pr} = 18 \cdot 24 + 0 \cdot 60 = 18 \cdot 84$ cm. A paraxial ray-trace (with initial angle u chosen arbitrarily) can then be made and the result of tracing this ray through the five surfaces of the eyepiece gives the position of the exit pupil as $1 \cdot 040$ cm. from the surface of the eye lens.

Through this position three parallel rays, *a*, *pr* and *b* (see Fig. 25) are then traced (in N_y light) at an angle to the axis corresponding to about two-thirds of the semi-field of view, namely about 12 degrees and with a semi-diameter of exit pupil of 0·20 cm. The method for carrying out oblique ray tracing is given in the section on photographic lenses (see page 89) which should be followed carefully before attempting this eyepiece ray tracing. The initial data for the calculation will be as follows :—

$$L_{pr} = -1\cdot040 \text{ cm.} \qquad U_{pr} = -12^\circ\text{–}0'\text{–}0''$$

$$L_a = L_{pr} + \text{SA.cotan}\,U_{pr} = -1\cdot040 + 0\cdot20.\text{cotan } 12^\circ = -1\cdot9809 \text{ cm.}$$

$$L_b = L_{pr} - \text{SA.cotan}\,U_{pr} = -1\cdot040 - 0\cdot20.\text{cotan } 12^\circ = -0\cdot0991 \text{ cm.}$$

The radii, axial thicknesses, air space and refractive indices N_y for the three lenses have already been given on page 68.

Calculation No. 17 shows the ray tracing and gives the following final values :—

$$U'_a = +5^\circ\text{–}59'\text{–}16'' ; \qquad U'_{pr} = +1^\circ\text{–}38'\text{–}54'' ; \qquad U'_b = -2^\circ\text{–}46'\text{–}27''$$

$$L'_a = +5\cdot6476 \text{ cm.} ; \qquad L'_{pr} = +18\cdot8777 \text{ cm.} ; \qquad L'_b = -10\cdot1866 \text{ cm.}$$

Using these values in the closing equations we get :—

$$L'_{ab} = L'_b - \frac{(L'_b - L'_a)\sin U'_a}{\sin(U'_a - U'_b)} \,.\, \cos U'_b = +0\cdot6444 \text{ cm.}$$

$$H'_{ab} = \frac{(L'_b - L'_a)\sin U'_a}{\sin(U'_a - U'_b)} \,.\, \sin U'_b = +0\cdot5248 \text{ cm.}$$

$$\text{Coma}'_T = (L'_{pr} - L'_{ab})\tan U'_{pr} - H'_{ab} = -0\cdot0001$$

and $\text{Coma}'_S = \frac{1}{3}\,\text{Coma}'_T = -\cdot00003.$

It is evident, therefore, that by the exact trigonometrical test the eyepiece has proved to be reasonably free from coma at a 12 degree semi-field, the coma'_S being slightly smaller than the permissible tolerance of $\pm\ 0\cdot0025$. We will now test trigonometrically the chromatic difference of magnification ; this may be done by comparing the value H'_{ab} given by the ray-tracing of Calculation No. 17 (which was for yellow-green light) with a similar ray-tracing for a and b rays in blue light. If this is done with N_v values for each of the glasses, the following results will be obtained :—

$$\text{Final } L'_a = +5\cdot5242 \text{ cm.} \qquad U'_a = +6^\circ\text{–}8'\text{–}3''$$

$$\text{Final } L'_b = -10\cdot6510 \text{ cm.} \qquad U'_b = -2^\circ\text{–}38'\text{–}57''$$

giving H'_{ab} (for blue light) $= +0\cdot5231$ cm.

and therefore, Chrom. Diff. of Mag. $= H'_{ab}$ (yellow-green) $- H'_{ab}$ (blue)

$$= +0\cdot5248 - 0\cdot5231$$
$$= +0\cdot0017 \text{ cm.}$$

Another point of interest which arises out of the oblique ray tracing we have carried out with this eyepiece, is that it enables us to get an idea of the extent of the distortion produced. This can be done by comparing the actual value of H'_{ab} (given by the ray tracing) with the ideal H'_k obtained from a knowledge of the equivalent focal length of the eyepiece (namely 2.485 cm.) and the initial semi-angular field, i.e., 12 degrees; for referring

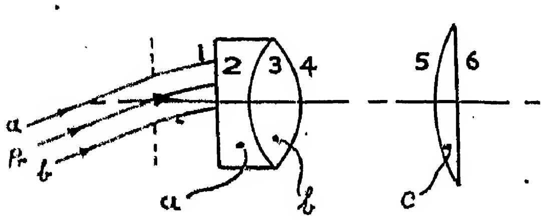


Fig. 25.

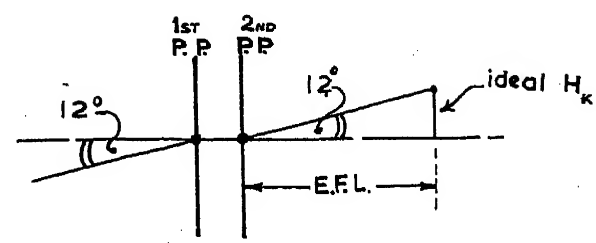


Fig. 25a.

to Fig. 25A an initial ray meeting the first principal point of the eyepiece system under an angle of twelve degrees will leave the second principal point under the same angle and therefore $\dfrac{\text{Ideal } H'_k}{\text{E.F.L.}} = \tan 12°$ so that

$$\text{Ideal } H'_k = 2\cdot485 \times \tan 12° = 0\cdot5283 \text{ cm.}$$

Now the actual H'_{ab} from the ray tracing was $0\cdot5248$ cm. and the Distortion may be expressed as the percentage ratio of this difference (namely $0\cdot0035$ cm.) to the ideal H'_k, that is $\dfrac{0\cdot0035}{0\cdot5283} \times 100 = 0\cdot6$ per cent. Alternately, it may be expressed as the ratio of the difference in angle that these two object heights would produce on emergence from the eye lens to the initial angle of twelve degrees. For example, the tangent of the angle subtended at the 2nd principal point of the eyepiece system by the actual H'_{ab} is equal to $\dfrac{0\cdot5248}{2\cdot485} = 0\cdot2112$ and the angle therefore is $11°$–$56'$ giving a difference of 4 minutes of arc from the ideal 12 degrees. Expressed as a percentage distortion we have $\dfrac{4}{12 \times 60} \times 100 = 0\cdot6$ per cent.

The advantage of the latter method of expressing the distortion, namely in percentage angular amount, is that it does not matter at what distance

the virtual image seen through the eyepiece is situated (i.e., whether the eye is accommodated to view the image at the Near Point or at any distance between the Near Point and Infinity).

For further particulars concerning the general trend in eyepiece design together with data of more advanced types of eyepiece, the reader is referred to "The Inverting Eyepiece and its Evolution" by E. Wilfred Taylor—*Journ. Sci. Instrs.*, Vol. 22, March 1945.

CHAPTER IV

THE DESIGN OF PHOTOGRAPHIC LENSES

IN order to appreciate the high quality of definition given by the modern photographic lens, it is desirable to consider what the requirements of the ideal lens are, and then to see how nearly such requirements can be fulfilled. An ideal photographic lens should have :—(i) no chromatic aberration, (ii) no spherical aberration, (iii) no coma, (iv) no astigmatism, (v) no distortion, (vi) a perfectly flat field, (vii) rapidity of exposure, i.e., a small F/ratio value, (viii) large depth of focus and (ix) large angular field.

It is impossible to design a lens which will satisfy all these conditions simultaneously ; and indeed, it is difficult to correct more than a few of these aberrations at one time. Moreover, as some of the requirements (e.g., rapidity of exposure and large depth of focus) are opposed to one another, it is necessary to arrange the design according to the purpose for which the particular lens is to be used.

Further, it must be remembered that the image has to be formed, with rare exceptions, on a *flat* surface (either on plate or film) and the quality of definition over the area of the latter is governed by what the designer considers his criterion for the diameter of the "discs of least confusion". In general practice it is considered desirable to keep these diameters down to 0·001 inch for extremely sharp definition, 0·004″ for "good" definition, and 0·010″ for "soft" definition.

In this section, numerical examples will be taken to illustrate the principles involved in the design of photographic lenses of the meniscus types, the symmetrical types, and the anastigmat forms.

Early Lenses

The earliest type of lens used in cameras was of bi-convex form, and gave poor results generally. Such a lens had to be reduced in aperture to about F/32 to give any satisfactory definition on the plate.

Wollaston's discovery (1812) for the improvement of definition by using a diaphragm at a suitable position in front of the lens, was of great importance ; for it can be shown that by moving the diaphragm with respect to the lens a position will be found at which the coma can be eliminated. Fig. 26 (a), (b) and (c) illustrates this point which can be proved both graphically and by ray tracing methods. This fact is a basic principle employed in the systematic design of photographic lenses. It will be noted that the figure depicts

a meniscus lens and not a bi-convex lens, this is due to the fact that the former type is in more general use than the latter on account of the flatter field obtainable with the meniscus lens. This type still persists in large numbers in simple types of cameras, and although the quality of definition is not particularly good beyond F/16, the small number of air-glass surfaces (namely, two) prevents reflected and scattered light from reaching the plate and consequently good contrast in the image is secured.

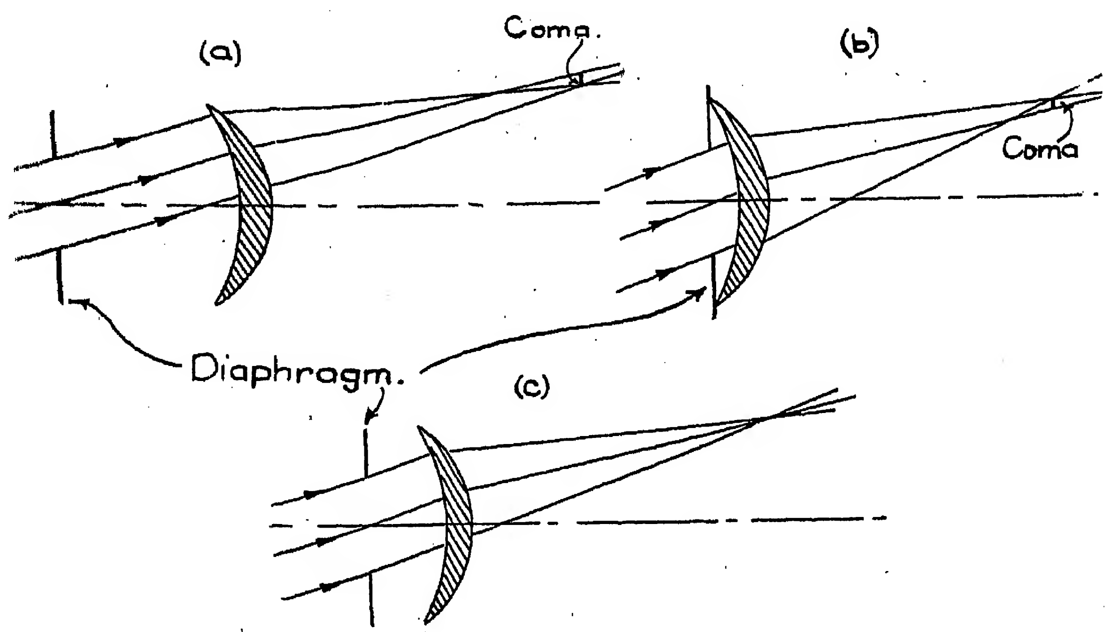


Fig. 26. Elimination of coma by movement of diaphragm.

We cannot do better, therefore, in the initial stages than to study in detail the way in which the general principles of photographic lens design may be applied to this particular type of lens, for the simplification introduced by having only two surfaces to deal with not only reduces the amount of work in the ray tracing but enables the resulting aberrations to be obtained in a relatively short time. Thus we may make some desired change in the lens system and be able to see the resulting effect more quickly than if a more complex lens were used. Hence, having put into practice the general methods for designing a lens of this form, the further steps necessary for dealing with the more advanced photographic lenses become much more readily understood.

Procedure of Design for a Simple "Landscape" Lens

(1) Using the specified focal length, determine the total curvature ($\mathcal{R}$) of the lens from $\frac{1}{f} = (N-1)\ \mathcal{R}$ for a number of types of glasses.

(2) With a constant power of lens for each glass, determine the Petzval Curvature (R_{ptz}) in each case. This will enable an idea to be gained of the most suitable glass which will give the *flattest* field.

(3) Having decided on the glass to be used, select three trial " bendings " or shapes of the lens.

(4) Trace a paraxial ray in G′-light parallel to the axis with an initial nominal $y = Y$ the semi-aperture, for each of the three shapes. Correct the second radius in each case to give the specified back focal length.

(5) Using the values obtained from the ray tracings of paragraph (4) determine the spherical aberration for each of the three lens shapes from the analytical formula for the spherical aberration contributed surface by surface as indicated in the SC′ equation given on page 79.

(6) Trace a principal ray with paraxial formulæ using an initial angle to the axis corresponding to the desired field for three different diaphragm positions, for each of the three shapes of lens.

(7) Utilising values obtained from the oblique ray tracings of paragraph (6) determine the amount of coma for each of the three stop positions for any one shape or " bending " of the lens. This may be done from the analytical formula for the coma contribution surface by surface (CC') given on page 79.

(8) The amount of coma is then plotted against the distance of the diaphragm or stop from one surface of the lens, and the stop distance found from the graph for zero amount of coma.

(9) To find the approximate amount of the astigmatism, Petzval curvature, and distortion from the respective analytical formulæ for AC', PC' and DC' given on page 79, it will be necessary to make a further oblique principal ray-trace commencing at the correct stop distance (l_{pr}) at which the lens is free from coma, in order to secure the values of i'_{pr} at every surface for use in these formulæ.

(10) Having determined the various aberrations by the analytical methods for each of the three " bendings " of the lens, all the results may then be summarised and a judgment formed as to the most suitable lens shape to employ compatible with the best definition over a flat plate.

(11) The most hopeful solution should then be tested by the strictly accurate trigonometrical methods, obtaining the *exact* amounts of spherical aberration, coma, astigmatism, curvature of field, and distortion.

(12) As the diameters of the " Discs of Least Confusion " are the criterion for the quality of definition given on the photographic plate, these must now be calculated for both the axial and oblique rays. Their positions must also be determined.

(13) The astigmatic fields, the curvature of field, the Petzval curvature, the lens, the diaphragm, and the principal ray emerging from the lens, may

then be drawn to scale for each lens-shape in turn. Thus a graphical picture of the performance of the lens may be obtained.

(14) In more advanced designs, it may be necessary to carry out the procedure mentioned in paragraphs (11), (12) and (13) for each of the three "bendings", for although the use of the analytical formulæ is a great saving in time and work; it only gives *approximate* results.

Simple Meniscus Lens

For the purpose of this example we will ask that a (so-called) "landscape" lens be designed having a back focal length of ten inches, an effective aperture of 0·6 inch, and to cover a plate whose diagonal dimension is six inches. (The lens is to be of one type of glass only.)

Commencing with the first part of the procedure we will choose (say) four glasses from the catalogue covering a fairly wide range of refractive indices, such as

	N_D	$N_{G'}$
Fluor Crown	1·4785	1·4869
Hard Crown	1·5155	1·5263
Dense Flint	1·6246	1·6501
Extra Dense Flint	1·7566	1·7938

We will assume that the photographically active rays are those for G′-light ($\lambda = 4340$A.) and will therefore use the refractive index value for G′ throughout the calculations. The total curvature ($\mathcal{R}$) of the lens can now be obtained for a focal length of ten inches from the usual thin lens equation $\frac{1}{f} = (N-1)\,\mathcal{R}$ and if we do this for the above-named four glasses, we get :—

Fluor Crown	$\mathcal{R} = 0{\cdot}205$
Hard Crown	$\mathcal{R} = 0{\cdot}190$
Dense Flint	$\mathcal{R} = 0{\cdot}154$
Extra Dense Flint	$\mathcal{R} = 0{\cdot}126$

The Petzval Curvature (R_{ptz}) may be obtained from $R_{ptz} = \frac{1}{r_{ptz}} = \Sigma \frac{(N'-N)}{r \,.\, N \,.\, N'}$ where N and N' are successively the refractive indices of the medium on the left and right respectively of each surface r. If we apply this relation to a single lens in air, then

$$\frac{1}{r_{ptz}} = \left(\frac{N'-N}{r_1 \,.\, N \,.\, N'}\right)_{\text{1st surface}} + \left(\frac{N'-N}{r_2 \,.\, N \,.\, N'}\right)_{\text{2nd surface}}$$

from which it will be obvious that in order to make $R_{ptz} =$ zero (i.e., to secure a flat field) the radii r_1 and r_2 would have to be similar in numerical value

and sign; and consequently the lens would have little or no power unless very thick. Hence the nearest approach to the fulfilment of the Petzval condition is to have a lens with radii as nearly similar in value and sign as the conditions of focal length will allow. This results in a meniscus form of lens being used. The decision as to whether the lens should be used with the concave or convex surface towards the incident light is governed by other conditions which we shall see later.

As an interesting preliminary the Petzval curvature of the lens, using the various glasses in turn, will be calculated. The shape of the lens for this calculation will be quite arbitrarily chosen, for the Petzval curvature will not be altered provided the *power* of the lens does not change; so that choosing the curvature R_1 of the first surface $= -0\cdot300$ throughout, the curvature R_2 of the second surface may be determined from $\mathcal{R} = R_1 - R_2$ for each particular glass, giving:—

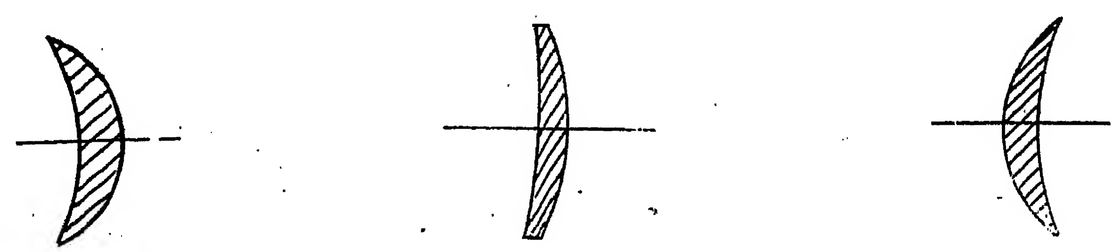
	R_1	r_1	R_2	r_2
Fluor Crown	$-0\cdot300$	$-3\cdot333''$	$-0\cdot505$	$-1\cdot980''$
Hard Crown	$-0\cdot300$	$-3\cdot333''$	$-0\cdot490$	$-2\cdot041''$
Dense Flint..	$-0\cdot300$	$-3\cdot333''$	$-0\cdot454$	$-2\cdot203''$
Extra Dense Flint ..	$-0\cdot300$	$-3\cdot333''$	$-0\cdot426$	$-2\cdot347''$

Using these radii, Calculation No. 18 shows the method of applying the Petzval formula given above. It should be pointed out, however, that if the lens is *thin*, the relation $r_{ptz} = N \, . \, f$ is valid; and Calculation No. 18 would not necessarily be required. It is given, nevertheless, in order to indicate the method when required later.

CALCULATION No. 18

	Fluor Crown		*Hard Crown*		*Dense Flint*		*Extra Dense Flint*	
	r_1	r_2	r_1	r_2	r_1	r_2	r_1	r_2
N' $-N$	1·4869 1·0000	1·0000 1·4869	1·5263 1·0000	1·0000 1·5263	1·6501 1·0000	1·0000 1·6501	1·7938 1·0000	1·0000 1·7938
$(N'-N)$	+0·4869	−0·4869	0·5263	−0·5263	0·6501	−0·6501	0·7938	−0·7938
$\log(N'-N)$ + colog N' + colog N + colog r	9·6874 9·2877 0·0000 9·4772n	9·6874n 0·0000 9·8277 9·7033n	9·7212 9·8164 0·0000 9·4772n	9·7212n 0·0000 9·8164 9·6902n	9·8130 9·7825 0·0000 9·4772n	9·8130n 0·0000 9·7825 9·6570n	9·8997 9·7462 0·0000 9·4772n	9·8997n 0·0000 9·7462 9·6295n
log sum	8·9923n	9·2184	9·0148n	9·2278	9·0727n	9·2525	9·1231n	9·2754
$\left(\frac{N'-N}{r\,.\,N\,.\,N'}\right)$	−0·0982	+0·1654	−0·1035	+0·1690	−0·1182	+0·1788	−0·1328	0·1885
$1/r_{ptz}$	+0·0672		+0·0655		+0·0606		+0·0557	
r_{ptz}	14·88″		15·27″		+16·50″		+17·95″	

It will be recalled that the Petzval curvature represents the curvature of field in the absence of astigmatism and is based on the assumption that the aperture of the lens is *small*. In practice it is exceedingly difficult to remove all traces of astigmatism and of course lens apertures are not necessarily small, but in spite of this hypothetically ideal nature of the Petzval theorem, it does supply certain information which is valuable at the outset of a design ; for example, in the particular case we are dealing with, it tells us that a flatter field (see Calculation No. 18 or from $r_{pt.} = N \cdot f$) may be expected by using a glass of high refractive index, and as mentioned before a *meniscus* form of lens is desirable.



$R_1 = -0{\cdot}500$
$R_2 = -0{\cdot}654$
$r_1 = -2{\cdot}000''$
$r_2 = -1{\cdot}529''$

$R_1 = -0{\cdot}10$
$R_2 = -0{\cdot}254$
$r_1 = -10{\cdot}000''$
$r_3 = -3{\cdot}937''$

$R_1 = +0{\cdot}400$
$R_2 = +0{\cdot}246$
$r_1 = +2{\cdot}500''$
$r_2 = +4{\cdot}065''$

Fig. 27.

(In order to visualise what such curvatures as fifteen or sixteen inches appear as, when limited to covering a plate of six inches across the diagonal it is well to draw such curves with a beam compass on squared paper, when the variation from a flat plate will be readily apparent.)

From the results of Calculation No. 18 let us therefore choose the dense flint glass with $N_{G'} = 1{\cdot}6501$ for the lens which is to be designed. From the total curvature ($\mathcal{R} = 0{\cdot}154$) we may now select three shapes of the lens from $\mathcal{R} = R_1 - R_2$. Taking R_1 equal to (say) $-0{\cdot}50$, $-0{\cdot}10$ and $+0{\cdot}40$ we get shapes of lens as indicated in Fig. 27.

From now on, we will take one only of the three shapes (say, the first) and work right through the necessary analytical procedure, bearing in mind that this must be carried out also for the other two.

In the initial stages of a design it may be desirable and prove a saving of time, to get an approximate idea of the extent of the various aberrations for the particular conditions concerned and if the latter seem hopeful then to follow this up by an exact determination of the aberrations by trigonometrical methods.

Dealing therefore with the approximate determinations first, the following

analytical formulæ (see Conrady's *Applied Optics*, page 314) will be used :—

Spherical Aberration Contribution (SC′)

$$SC' = \tfrac{1}{2}\, l' \,.\, u' \,.\, N' \,.\, i'(i' - u)(i - i')/N'_k \,.\, u'^2_k$$

or for a Plane Surface

$$SC' = \tfrac{1}{2}\, l' u'^2 (u' + u)(u' - u) \,.\, N'/N'_k \,.\, u'^2_k$$

Coma Contribution (*CC′*)

$$CC' = SC' \,.\, u'_k \,.\, i'_{pr}/i'$$

Astigmatism Contribution (*AC′*)

$$AC' = SC' \left(\frac{i'_{pr}}{i'}\right)^2$$

Petzval Curvature (*PC′*)

$$PC' = \tfrac{1}{2}\, H'^2_k \,.\, N'_k\, (N' - N)/N \,.\, N' \,.\, r$$

in which $H'_k = -(l'_k - l'_{pr_k}) u'_{pr_k}$

Distortion Contribution (*DC′*)

$$DC' = CC' \left(\frac{i'_{pr}}{i'}\right)^2 + PC' \,.\, u'_k \,.\, \frac{i'_{pr}}{i'}$$

These formulæ give the various aberrations contributed surface by surface and have to be calculated accordingly, and in order to do this it is necessary to use the values from a paraxial-ray trace parallel to the axis and a paraxial principal-ray trace at the particular angle to the axis necessary to cover the desired field.

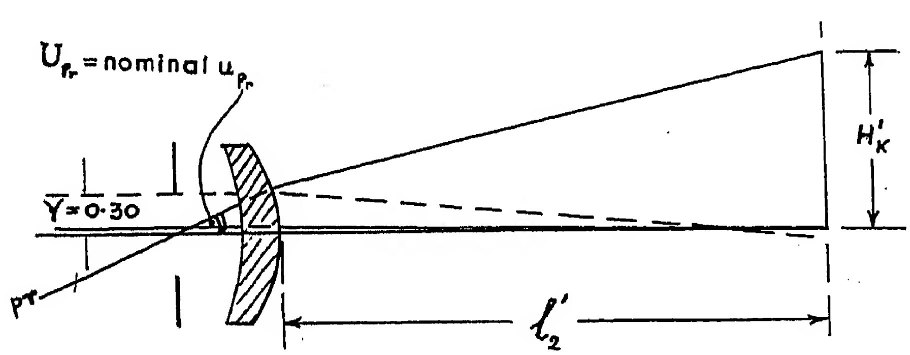


Fig. 28.

Treating the *SC′* first, we must therefore trace a paraxial ray with normal $y = Y = 0 \cdot 300$ through the first shape of the lens (see Fig. 28). As the lens is "thick" the radius ($r_2 = -1 \cdot 529$) given by the "thin" lens formula for the power of the lens will be found to give a widely different value of l'_2 from the specified 10 inches, and therefore it is desirable to correct r_2 after the paraxial ray has been traced through the first surface using the familiar formula for refraction at one single surface, i.e., $\dfrac{N'}{l'} - \dfrac{N}{l} = \dfrac{N' - N}{r}$.

Applying this we find that r_2 should be $-1\cdot644''$ and the values obtained from the ray trace for use in the analytical spherical aberration contribution formula are as follows :—

	1st surface	2nd surface
l'	$-5\cdot0764$	$+10\cdot0475$
u'	$-0\cdot059097$	$+0\cdot032799$
i'	$-0\cdot090903$	$-0\cdot233254$
u	$0\cdot000000$	$-0\cdot059097$
	$u'_k = +0\cdot032799$	

and Calculation No. 19 shows the method of using the SC' formula on page 79.

CALCULATION No. 19

	1st surface	2nd surface
$\log \frac{1}{2}$	9·6990	9·6990
$+ \log l'$	0·7056*n*	1·0021
$+ \log u'$	8·7716*n*	8·5159
$+ \log N'$	0·2175	0·0000
$+ \log i'$	8·9586*n*	9·3678*n*
$+ \log (i'-u)$	8·9586*n*	9·2409*n*
$+ \log (i-i')$	8·7716*n*	8·9633
$+ 2 \operatorname{colog} u'$	2·9683	2·9683
$\log SC'$	9·0508*n*	9·7573
SC'	$-0\cdot1124$	0·5719

Total Spherical Aberration $= +0\cdot4595''$.

We must now proceed with the important point of eliminating the coma by finding the correct position of the stop or diaphragm. This may be done by employing the coma contribution formula given above for three different positions of the stop and then plotting the latter against the amount of the coma in each case. Where this line crosses the abscissa the *best* position of the diaphragm will be found.

Referring to the CC' equation we see that the only term which is as yet unknown is the value i'_{pr} at each surface; to obtain these it is necessary to trace a principal ray (pr) through the centre of the stop at the required inclination to the axis, i.e., the angle u_{pr} in Fig. 28. Taking this angle as the natural number corresponding to log sine 16°–42′–0″ in order to give a final H'_k equal to approximately three inches this oblique principal ray must now be traced for two stop-positions chosen quite arbitrarily as (say) $-0\cdot30$ and $-0\cdot70$ inches. Calculation No. 20 shows these tracings :—

CALCULATION No. 20

Oblique principal ray trace.

	1st Surface		2nd Surface	
Stop distance	$-0\cdot30$	$-0\cdot70$	$-0\cdot30$	$-0\cdot70$
l $-r$	$-0\cdot300$ $+2\cdot000$	$-0\cdot700$ $+2\cdot000$	$-0\cdot951$ $+1\cdot644$	$-1\cdot441$ $+1\cdot644$
$(l-r)$	$+1\cdot700$	$+1\cdot300$	$+0\cdot693$	$+0\cdot203$
$\log u$ $+\log(l-r)$	$9\cdot45843n$ $0\cdot23045$	$9\cdot45843n$ $0\cdot11394$	$9\cdot28133n$ $9\cdot84073$	$9\cdot32996n$ $9\cdot30750$
$\log(l-r)u$ $-\log r$	$9\cdot68888n$ $0\cdot30103n$	$9\cdot57237n$ $0\cdot30103n$	$9\cdot12206n$ $0\cdot21590n$	$8\cdot63746n$ $0\cdot21590n$
$\log i$ $+\log\left(\frac{N}{N'}\right)$	$9\cdot38785$ $9\cdot78249$	$9\cdot27134$ $9\cdot78249$	$8\cdot90616$ $0\cdot21751$	$8\cdot42156$ $0\cdot21751$
$\log i'$ $+\log r$	$9\cdot17034$ $0\cdot30103n$	$9\cdot05383$ $0\cdot30103n$	$9\cdot12367$ $0\cdot21590n$	$8\cdot63907$ $0\cdot21590n$
$\log r\,.\,i'$ $-\log u'$	$9\cdot47137n$ $9\cdot28133n$	$9\cdot35486n$ $9\cdot32996n$	$9\cdot33957n$ $9\cdot38651n$	$8\cdot85497n$ $9\cdot36349n$
$\log(l'-r)$	$0\cdot19004$	$0\cdot02490$	$9\cdot95306$	$9\cdot49148$
u $+i$	$-0\cdot287363$ $0\cdot244259$	$-0\cdot287363$ $0\cdot186784$	$-0\cdot191131$ $0\cdot080568$	$-0\cdot213775$ $0\cdot026397$
$u+i$ $-i'$	$-0\cdot043104$ $-0\cdot148027$	$-0\cdot100579$ $-0\cdot113196$	$-0\cdot110563$ $-0\cdot132944$	$-0\cdot187378$ $-0\cdot043558$
u'	$-0\cdot191131$	$-0\cdot213775$	$-0\cdot243507$	$-0\cdot230936$
$(l'-r)$ $+r$	$1\cdot5490$ $-2\cdot000$	$1\cdot0590$ $-2\cdot000$	$0\cdot8976$ $-1\cdot644$	$0\cdot3101$ $-1\cdot644$
l' $-d'$	$-0\cdot4510$ $-0\cdot50$	$-0\cdot9410$ $-0\cdot50$	$-0\cdot7464$	$-1\cdot3339$
new l	$-0\cdot9510$	$-1\cdot4410$		

Utilising the values above of log i'_{pr} for the first surfaces as $9\cdot1703$ and $9\cdot0538$ and those for the second surfaces as $9\cdot1237$ and $8\cdot6391$ in the CC' formula, we find from Calculation No. 21 that the total Coma$'_s$ for stop position $=-0\cdot30$ is $-0\cdot0049$ and for stop position $=-0\cdot70$ is $+0\cdot0011$. (It is generally advisable to take a third stop distance; and if, for example, $-0\cdot50$ is taken, the coma will be found to be $-0\cdot0018$.)

These are plotted as ordinates against the stop distance as abscissa (see Fig. 29) and the position for zero coma gives a diaphragm position of $-0{\cdot}61$ inches from the front surface of the lens.

CALCULATION No. 21

$$CC' = SC' \cdot u'_k \cdot \frac{i'_{pr}}{i'}$$

	Stop distance $= -0{\cdot}30$		Stop distance $= -0{\cdot}70$	
	1st Surface	2nd Surface	1st Surface	2nd Surface
$\log SC'$	9·0508*n*	9·7573	9·0508*n*	9·7573
$+ \log u'_k$	8·5159	8·5159	8·5159	8·5159
$+ \log i'_{pr}$	9·1703	9·1237	9·0538	8·6391
log numerator	6·7370*n*	7·3969	6·6205*n*	6·9123
$- \log i'$	8·9586*n*	9·3678*n*	8·9586*n*	9·3678*n*
$\log CC'$	7·7784	8·0291*n*	7·6619	7·5445*n*
CC'	0·0060(0)	− 0·0010(7)	0·0045(9)	− 0·0035(0)

Total $\text{Coma}'_s = -0{\cdot}0049$ Total $\text{Coma}'_s = +0{\cdot}0011$.

Another principal ray must then be traced for this "best stop position" in order to furnish the correct values for i'_{pr} and u'_{pr_k} for use in the remaining analytical formulæ in determining the Astigmatism (AC'), the Petzval Curvature (PC'), and the Distortion (DC'). At the same time it serves as a

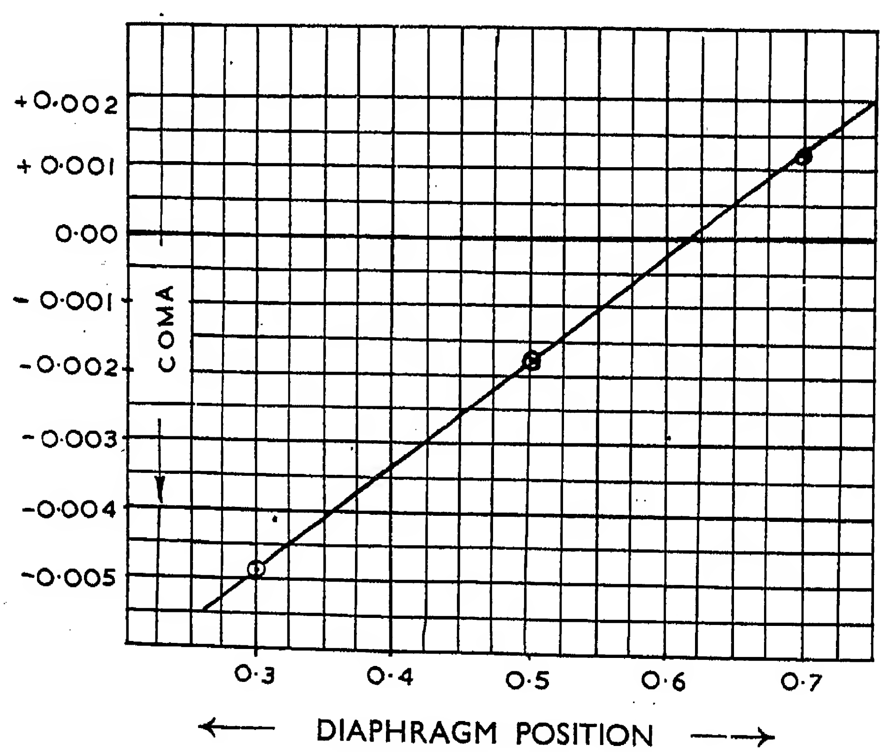


Fig. 29.

check on the two previous oblique ray tracings, for as coma plots as a straight line the resulting total Coma'_s obtained from the third oblique tracing should give a point on the graph which is in a straight line with the other two, and should of course be zero or very nearly so.

The values for log i'_{pr} from this third ray tracing will be found to be : (1st surface) $= 9\cdot08291$ and (2nd surface) $= 8\cdot80396$; whilst $u'_{pr_k} = -0\cdot233765$ and $l'_{pr_k} = -1\cdot1962$. Thus, with a stop distance of $-0\cdot61$ inches the total Coma'_s will be found to be $-0\cdot0002$. This value is now so small that the coma can be considered as being corrected.

We will now go on to the AC' calculation, the method being shown in Calculation No. 22. It should be pointed out that the value for AC' given by this formula, gives the distance of the sagittal focal surface from the Petzval surface. But as the distance of the tangential focus from the Petzval surface bears a fixed relation of 3 to 1 to the sagittal distance, it follows that the value for the actual amount of the astigmatism will be twice the AC' value.

Calculation No. 22

$$AC' = SC'\left(\frac{i'_{pr}}{i'}\right)^2$$

	1st Surface	2nd Surface
$\log i'_{pr}$ $-\log i'$	$9\cdot0829$ $8\cdot9586n$	$8\cdot8040$ $9\cdot3678n$
$\log\left(\frac{i'_{pr}}{i'}\right)$	$0\cdot1243n$	$9\cdot4362n$
$2\log\left(\frac{i'_{pr}}{i'}\right)$ $+\log SC'$	$0\cdot2486$ $9\cdot0508n$	$8\cdot8724$ $9\cdot7573$
$\log AC'$	$9\cdot2994n$	$8\cdot6297$
AC'	$-0\cdot1993$	$+0\cdot0426$

Total $AC' = -0\cdot1567$.

Regarding the Petzval Curvature, this has already been calculated (see page 77) when various glasses were being tried, but if this had *not* been done the analytical formula for the PC' would automatically be used. The latter, however, will be used here in order to illustrate the method ; the calculation (No. 23) involves first the determination of H'_k from

$H'_k = -(l'_k - l'_{pr_k})\,u'_{pr_k}$ and then the use of the equation already given for PC' on page 79.

Calculation No. 23

$$PC' = \frac{1}{2} H'^2_k \,.\, N'_k \,.\, \frac{(N'-N)}{N.N'.r} \text{ in which } H'_k = -(l'_k - l'_{pr_k})\,u'_{pr_k}$$

	1st surface	2nd surface
$\log N$ $+\log N'$ $+\log r$	0·0000 0·2175 0·3010n	0·2175 0·0000 0·2159n
log denominator	0·5185n	0·4334n
$\log \frac{1}{2}$ $+ 2\log H'_k$ $+\log N'_k$ $+\log(N'-N)$	9·6990 0·8394 0·0000 9·8130	9·6990 0·8394 0·0000 9·8130n
log numerator − log denominator	0·3514 0·5185n	0·3514n 0·4334n
$\log PC'$	9·8329n	9·9180
PC'	−0·6806	+0·8279

Determination of H'_k	
l'_k $-l'_{pr_k}$	10·0475 −(−1·1962)
$(l'_k - l'_{pr_k})$	+11·2437
$\log -(l'_k - l'_{pr_k})$ $+\log u'_{pr_k}$	1·05091n 9·36878n
$\log H'_k$	0·41969
H'_k	2·628″

Total $PC' = +\,0·1473$.

It will be noticed that this calculation gives the Petzval curvature in the form of a distance along the axis corresponding to a definite height of image H'_k (see Fig. 29a); if, however, this distance, namely 0·1473, is converted into radius of curvature (by the ordinary properties of the circle) knowing H'_k, the r_{ptz} will be found to be 23·52 inches.

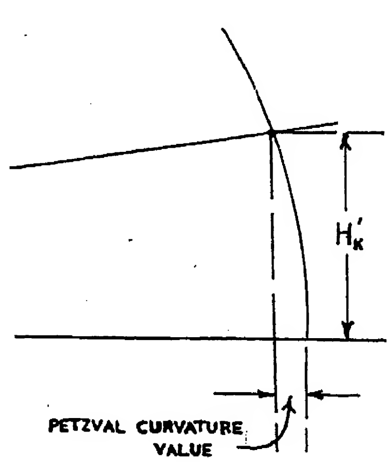


Fig. 29a.

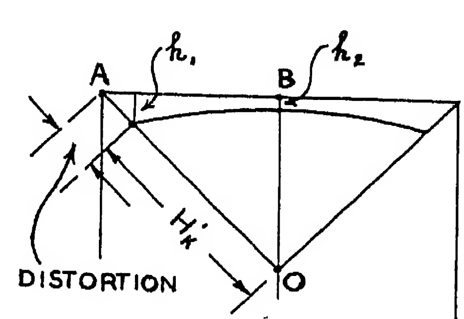


Fig. 30.

Lastly, the Distortion will be determined. This is shown in Calculation No. 24.

CALCULATION No. 24

$$DC' = CC'\left(\frac{i'_{pr}}{i'}\right)^2 + PC' \,.\, u'_k \,.\, \frac{i'_{pr}}{i'}$$

	1st Term	
	1st surface	*2nd surface*
log CC'	7·6910	7·7094n
$+ 2\log\left(\frac{i'_{pr}}{i'}\right)$	0·2486	8·8724
log 1st term	7·9396	6·5818n
antilogs	+0·0087	−0·0004

	2nd Term	
	1st surface	*2nd surface*
log PC'	9·8329n	9·9180
+ log u'_k	8·5159	8·5159
$+ \log\left(\frac{i'_{pr}}{i'}\right)$	0·1243n	9·4362n
log 2nd term	8·4731	7·8701n
antilogs	+0·0297	−0·0074

1st term = + 0·0083

2nd term = + 0·0223

∴ Total $DC' = + 0{\cdot}0306$.

Referring to Fig. 30 we may determine the type of the distortion, namely whether it is " barrel " or " pincushion ". As distortion varies as $H_k'^3$, it follows that the height h_2 will be equal to $\left(\frac{\text{OB}}{\text{OA}}\right)^3 \times 0{\cdot}0306$. But OA $= H'_k + 0{\cdot}0306 = 2{\cdot}628$ (see Calculation No. 23) $+ 0{\cdot}0306 = 2{\cdot}6586$; and OB $=$ OA $\cos 45° = 2{\cdot}6586 \times 0{\cdot}7071 = 1{\cdot}880$; so that $h_2 = \left(\frac{1{\cdot}880}{2{\cdot}628}\right)^3 \times 0{\cdot}0306 = 0{\cdot}0112$. The height $h_1 = 0{\cdot}0306 \times \cos 45° = 0{\cdot}0216$ It will be seen that h_2 is smaller than h_1, and it therefore follows that the distortion is of the " barrel " type. If the foregoing set of calculations are now repeated for the second and third bendings in turn, the values given respectively in lines 2 and 3 of Table V will be found.

A survey of the results given in Table V would at first sight indicate that bending No. 3 was the best of the three solutions on account of the low values for the various aberrations, more especially the astigmatism ; but before one can assess the results correctly the shape of the astigmatic fields must be obtained, and the true curvature of field determined with respect to the flat plate.

TABLE No. V

Shape or Bending	*Stop Distance with respect to First Surface*	*Spherical Aberration*	*Coma$'_s$*	*AC′ Value*	*Astigmatism (actual)*	*Petzval sum and radius*	*Distortion*
No. 1	−0·61″	+0·4595	−0·0002	− ·01567	−0·3134	+0·1473 $r_{ptz}=23\cdot52$	+0·0302
No. 2	−2·17″	+0·0707	−0·0003	−0·0529	−0·1058	+0·2348 $r_{ptz}=16\cdot62$	+0·0459
No. 3	+2·09″	+0·0806	−0·0016	−0·0076	−0·0152	+0·2295 $r_{ptz}=21\cdot32$	−0·0614

This will now be carried out to scale. Referring to Fig. 31 (1st bending) we set out on the drawing board the main axis and draw the lens with its correct radii and axial thickness to scale. Measure off from the pole A of the second surface a distance $l'_k = 10\cdot0475$ inches (10·05″ will be the nearest we can measure to) and at the point I erect a perpendicular of height $H'_k = 2\cdot628''$ (as determined in the PC′ calculation). Then set off a distance $l'_{pr_k} = -1\cdot196''$ to the *left* of A giving the point B. Join this point to the H'_k height on the perpendicular when BE will give the inclination to the axis of the oblique principal ray. With a beam compass set to the radius of the Petzval curvature (namely 23·52″) draw this circle through the point I; where it cuts BE in P, set off along BE a distance PS equal to 0·1567″ (the value given by the AC' calculation). It has been pointed out that the AC' calculation gives the distance of the sagittal focal surface from the Petzval surface, and as the tangential focus bears a fixed ratio of 3 to 1 to the former distance (under conditions of primary aberration), we may set off a distance PT equal to three times 0·1567 or 0·4701″.

The astigmatic surfaces SI and TI may then be drawn. Lastly the true curvature of field may be put in; this is the locus of points mid-way between the sagittal and tangential foci for all inclinations from the axis, so that bisecting the distance ST in C the line CI represents the curvature of the field.

Finally, a straight line perpendicular to the main axis is drawn from the point I; this will represent the flat photographic plate, and the curvature of field with respect to this may be clearly judged.

The other diagrams of Fig. 31 show the foregoing procedure carried out and carefully drawn for the other two bendings or shapes of the lens.

Examination of the three diagrams indicates that the curvature of field is bending away from the lens for its first shape, curved towards the lens for

the second, and still curved towards the lens for the third. Thus a shape of lens intermediate between that of the first and second would give the best solution for obtaining a *flat* field.

Let us choose therefore a shape of lens with $r_1 = -3\cdot333''$ and $r_2 = -2\cdot304''$ as a possible " best " solution ; in this case, however, instead of using the

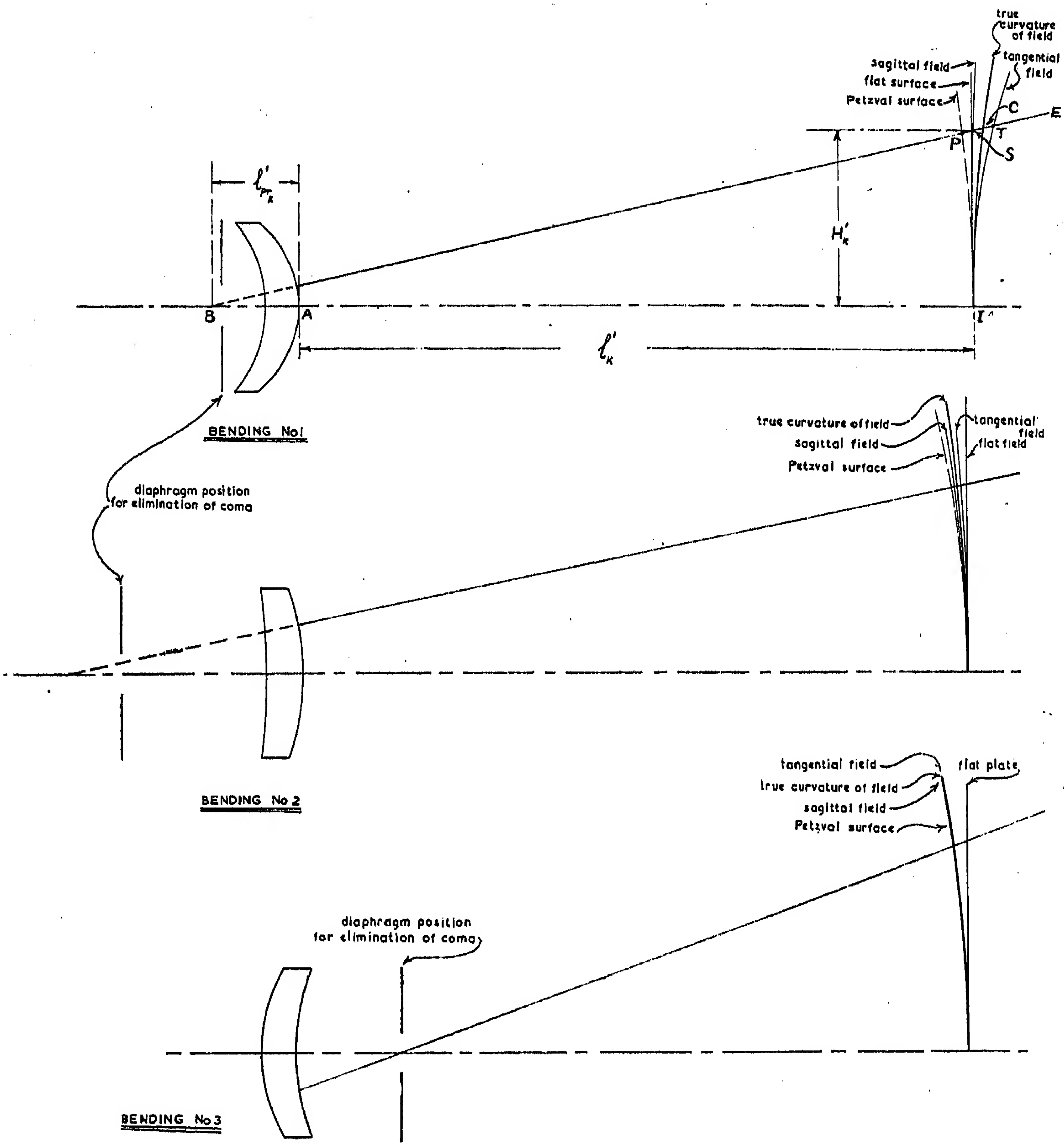


Fig. 31. Scale $\frac{1}{3}$.

analytical methods for determination of the aberrations, we will test the solution by the trigonometrically exact method. Thus, the procedure involved in the latter will be numerically illustrated and the exact values of the aberrations obtained.

The first thing to do will be to test the spherical aberration; this will be done by tracing a marginal ray at the semi-aperture height $Y = 0 \cdot 30$ and a paraxial ray with a similar nominal value both in G′-light, ($N_{G'} = 1 \cdot 6501$). The result of this ray-tracing will be found to be as follows:—

Final $l' = 10 \cdot 2041$ Final $u' = + 0 \cdot 031137$
Final $L' = 9 \cdot 9845$ Final $U' = + 1°\text{–}49'\text{–}2''$

giving the Spherical Aberration $(l' - L') = + 0 \cdot 2196''$.

The next part of the procedure is to find the correct position of the diaphragm for the elimination of coma. This may be done in the first instance by means of the method already described after having traced a principal ray at three different stop positions. The position of the diaphragm for zero coma will be found to be $- 1 \cdot 14$ inches from the first surface of the lens, but this will now be tested trigonometrically by tracing three rays.

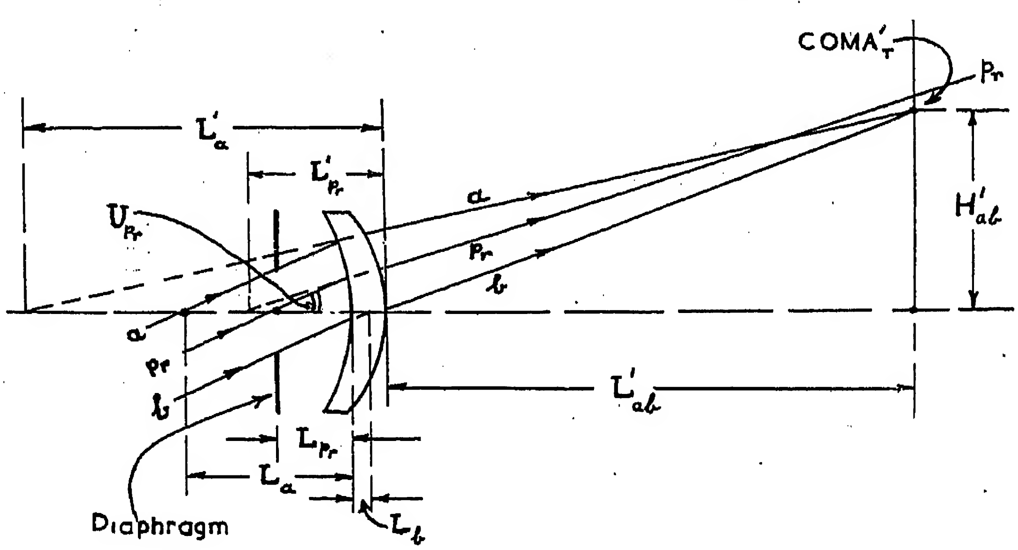


Fig. 32.

"a", "pr", and "b" (see Fig. 32) obliquely through the lens. In the diagram coma will be indicated by the intersection of the "a" and "b" rays either above or below the principal ray "pr". This will be the coma due to the tangential rays passing through the lens, and will be called Coma'_T.

In order to commence the ray tracing we require to know L_a and L_b (see Fig. 32) which can be obtained as follows:—

$$L_a = L_{pr} + SA \text{ . cotan } U_{pr}$$
$$L_{pr} = - 1 \cdot 140'' \text{ (namely the stop distance)}$$
$$L_b = L_{pr} - SA \text{ . cotan } U_{pr}$$

where SA is the semi-aperture (i.e., $0 \cdot 30''$) and U_{pr} the initial inclination of

all three rays to the axis. Taking $U_{pr} = -16°–42'–0''$, we find $L_a = -2\cdot140$, and $L_b = -0\cdot140$. The ray-tracing for these three oblique rays is shown in Calculation No. 25.

CALCULATION No. 25

	First Surface			*Second Surface*		
	Ray *a*	Ray *pr*	Ray *b*	Ray *a*	Ray *pr*	Ray *b*
L $-r$	−2·140 +3·333	−1·140 +3·333	−0·140 +3·333	−2·9959 +2·304	−2·0518 +2·304	−0·7304 +2·304
$(L-r)$	+1·193	+2·193	+3·193	−0·6919	+0·2522	+1·5736
log sin U + log $(L-r)$	9·45843*n* 0·07664	9·45843*n* 0·34104	9·45843*n* 0·50420	9·39478*n* 9·84004*n*	9·33124*n* 9·40174	9·25339*n* 0·19689
log $(L-r)$ sin U − log r	9·53507*n* 0·52284*n*	9·79947*n* 0·52284*n*	9·96263*n* 0·52284*n*	9·23482 0·36248*n*	8·73298*n* 0·36248*n*	9·45028*n* 0·36248*n*
log sin I + log $\left(\frac{N}{N'}\right)$	9·01223 9·78249	9·27663 9·78249	9·43979 9·78249	8·87234*n* 0·21751	8·37050 0·21751	9·08780 0·21751
log sin I' + log r	8·79472 0·52284*n*	9·05912 0·52284*n*	9·22228 0·52284*n*	9·08985*n* 0·36248*n*	8·58801 0·36248*n*	9·30531 0·36248*n*
log r . sin I' − log sin U'	9·31756*n* 9·39478*n*	9·58196*n* 9·33124*n*	9·74512*n* 9·25339*n*	9·45233 9·30264*n*	8·95049*n* 9·36039*n*	9·66779*n* 9·41148*n*
log $(L'-r)$	9·92278	0·25072	0·49173	0·14969*n*	9·59010	0·25631
U $+I$	−16–42– 0 + 5–54–13	−16–42– 0 +10–53–55	−16–42– 0 +15–58–45	−14–22–13 − 4–16–28	−12–22–51 + 1–20–41	−10–19–28 + 7– 1–51
$U+I$ $-I'$	−10–47–47 − 3–34–26	− 5–48– 5 − 6–34–46	− 0–43–15 − 9–36–13	−18–38–41 + 7– 3–52	−11– 2–10 − 2–13–10	− 3–17–37 −11–39–10
U'	−14–22–13	−12–22–51	−10–19–28	−11–34–49	−13–15–20	−14–56–47
$L'-r$ $+r$	+0·8371 −3·333	+1·7812 −3·333	+3·1026 −3·333	−1·4115 −2·304	+0·3891 −2·304	+1·8043 −2·304
L' $-d'$	−2·4959 −0·500	−1·5518 −0·500	−0·2304 −0·500	−3·7155	−1·9149	−0·4997
new L	−2·9959	−2·0518	−0·7304			

Thus we find :—

$$L'_a = -3\cdot7155\ ; \qquad L'_{pr} = -1\cdot9149\ ; \qquad L'_b = -0\cdot4997$$

and $U'_a = -11°–34'–49''$; $U'_{pr} = -13°–15'–20''$; $U'_b = -14°–56'–47''$.

The closing equations for converting these values into L'_{ab} (see Fig. 32), H'_{ab}, and hence the coma'_T are as follows :—

$$L'_{ab} = L'_b - \frac{(L'_b - L'_a)\sin U'_a}{\sin (U'_a - U'_b)} \cdot \cos U'_b$$

$$H'_{ab} = \frac{(L'_b - L'_a)\sin U'_a}{\sin (U'_a - U'_b)} \cdot \sin U'_b$$

and $$\text{Coma}'_T = (L'_{pr} - L'_{ab}) \tan U'_{pr} - H'_{ab}.$$

Utilizing the above values in these equations, L'_{ab} will be found to be $10\cdot1226''$, $H'_{ab} = 2\cdot8356''$, and the $\text{Coma}'_T = +\,0\cdot0001$.

As the Coma'_S is one third Coma'_T, $\text{Coma}'_S = +\,0\cdot00003$.

Hence, it is obvious that the coma has been almost entirely eliminated by using the diaphragm at $1\cdot14''$ from the front surface of the lens.

The astigmatism will now be determined. This may be done by means of the so-called little "s" and little "t" formulæ, derived theoretically in Conrady's *Applied Optics* (pp. 407–413). The practical procedure for using these formulæ, however, is as follows :—

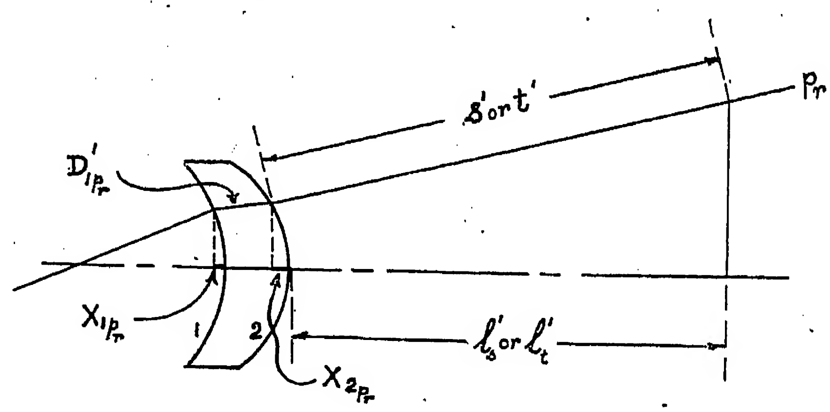


Fig. 33.

1. Employ the values obtained in the principal ray trace through the lens, for the best stop distance and particular inclination concerned.

2. Calculate the depth of curvature X_{pr} (see Fig. 33) corresponding to its point of incidence at each surface, from :—

$$X_{pr} = \frac{PA^2_{pr}}{2r} \text{ if } PA\text{-check has been used.}$$

or $$X_{pr} = 2r \,.\, \sin^2 \frac{U+I}{2} \text{ if } PA\text{-check has not been used.}$$

3. With these X values for each lens (or air space) of axial thickness d'_1, d'_2, the ray-length D'_{1pr}, D'_{2pr}, may be determined from :—

$$D'_{1pr} = (d'_1 + X_{2pr} - X_{1pr}) \,.\, \sec U'_{1pr}$$
$$D'_{2pr} = (d'_2 + X_{3pr} - X_{2pr}) \,.\, \sec U'_{2pr}$$

4. Now calculate for each surface,

$$s' = \frac{N'}{\left(\dfrac{N' \,.\, \cos I'_{pr} - N \,.\, \cos I_{pr}}{r} + \dfrac{N}{s}\right)}$$

and $$t' = \frac{N' \,.\, \cos^2 I'_{pr}}{\left(\dfrac{N' \cos I'_{pr} - N \cos I_{pr}}{r} + \dfrac{N \cos^2 I_{pr}}{t}\right)}$$

(N.B.—For infinitely distant objects the initial s and t will be ∞.)

5. The transfer equations from one surface to the next, will be :—

$$\left.\begin{aligned} s_{2pr} &= s'_{1pr} - D'_{1pr} \\ s_{3pr} &= s'_{2pr} - D'_{2pr} \end{aligned}\right\} \quad \text{and} \quad \left\{\begin{aligned} t_{2pr} &= t'_{1pr} - D'_{1pr} \\ t_{3pr} &= t'_{2pr} - D'_{2pr} \end{aligned}\right.$$

6. The final values for s' and t' after leaving the last surface k of the lens system are then expressed in terms of rectangular co-ordinates (l'_s and l'_t) about the optical axis (see Fig. 33) and therefore final $l'_s = s'_k \,.\, \cos U'_{pr_k} + X_{pr_k}$ and final $l'_t = t'_k \,.\, \cos U'_{pr_k} + X_{pr_k}$. And the true astigmatic difference of focus $= l'_s - l'_t$. N.B.—Astigmatism in terms of quality of definition on the plate $=$ (Extreme difference of focus) $\times$ tan U'_M.

Making use, therefore, of the foregoing procedure we will now determine the exact amount of the astigmatism given by our lens for the particular inclination of the rays to the axis, namely that given by the principal ray already traced in Calculation No. 25.

The astigmatism calculation is shown in full in No. 26.

Having determined the astigmatism at this particular inclination to the axis (namely 13°–15′–20″) and the positions of the sagittal and tangential foci, the true curvature of field must now be determined. In order to do this, it is necessary to locate the "discs of least confusion" both for the axial and oblique rays.

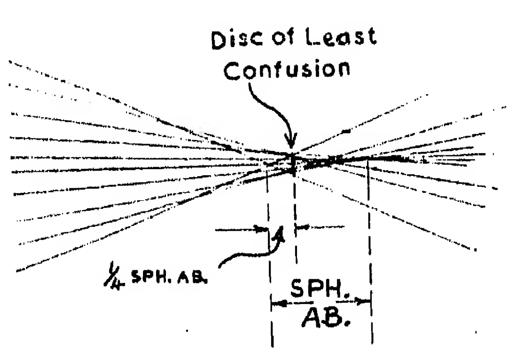


Fig. 34.

Dealing with the axial rays, first the position of the D.L.C.* (when spherical aberration is present) is situated at a distance from the *marginal ray focus* equal to one quarter of the spherical aberration (LA') see Fig. 34.

* D.L.C. the Disc of Least Confusion represents the direct geometrical attack on the problem of tolerances. Studies of the aberrational diffraction pattern often suggest wider tolerances.

Hence the position of the D.L.C. (from the last surface of the lens) $= L'_M + {}_4LA' = 9 \cdot 9845 + 0 \cdot 0549 = 10 \cdot 0394''$.

The position of the D.L.C. for the oblique rays is determined as follows :—

If there was no spherical aberration present, the D.L.C. would lie mid-way between the l'_s and l'_t positions. But as there is spherical aberration (amounting to $+0 \cdot 2196''$) present, we take the position on the oblique principal ray where the full-aperture rays focus, namely the distance L'_{ab}, and add one quarter of the spherical aberration. This will give the position of the D.L.C. for the tangential rays, and if we subtract the amount of astigmatism from this value the position of the D.L.C. for the sagittal rays will be obtained; and the mid-point of these two positions will give the D.L.C. position for the true curvature of field. For example: $L'_{ab} = 10 \cdot 1226$ and one quarter $LA' = 0 \cdot 0549$. Position of D.L.C. (tangential rays) $= 10 \cdot 1226 + 0 \cdot 0549 = 10 \cdot 1775$. Position of D.L.C. (sagittal rays) $= 10 \cdot 1775 -$ astigmatism $= 10 \cdot 1775 - 0 \cdot 2256 = 9 \cdot 9519$. So that the position of D.L.C. (for true curvature of field) $= \dfrac{10 \cdot 1775 + 9 \cdot 9519}{2} = 10 \cdot 0647$.

These positions may now be drawn out to scale (see Fig. 35) on the drawing board. From the pole A of the second surface of the lens, a distance equal to $10 \cdot 0394''$ ($10 \cdot 04$ as near as we can measure) is set off along the axis; this gives the position of the D.L.C. for the axial rays. Then set off from A a distance $L'_{ab} = 10 \cdot 12''$ and from this point a height $H'_{ab} = 2 \cdot 84''$ perpendicular to the axis. To the left of A, the distance $L'_{pr} = -1 \cdot 91''$ is marked off and this point joined to the H'_{ab} height, thus giving the inclination to the axis of the emergent principal ray. Along the axis from A, set off distances equal to $10 \cdot 18''$, $9 \cdot 95''$, and $10 \cdot 06''$ and erect perpendiculars; where these cut the oblique ray the respective positions of the tangential field, the sagittal field, and the true curvature of field, will be found.

It is desirable, in the case of a possible final solution, to determine the astigmatism for at least one other inclination to the axis, generally at two-thirds of the extreme inclination which covers the maximum size of plate. If this is done it will be found that for an emergent ray inclined at (say) 9 degrees to the axis the sagittal and tangential D.L.C. will lie on fairly regularly shaped astigmatic surfaces as shown in Fig. 35.

Whilst this may be so in the case of the simple form of lens we are considering at the moment; in more complex types of lens the astigmatic surfaces will not necessarily take a regular form of curve as above, indeed in the case of anastigmatic lenses the irregular shape of the curves and the varying amounts of astigmatism make it necessary to employ a number of inclinations in the calculations.

Having plotted the shape of the astigmatic fields (Fig. 35) and having drawn in the true curvature of field, it will be seen that the latter is very

nearly a straight line perpendicular to the axis, and therefore, this particular shape of the lens gives us the desired aim of attempting to obtain a flat field.

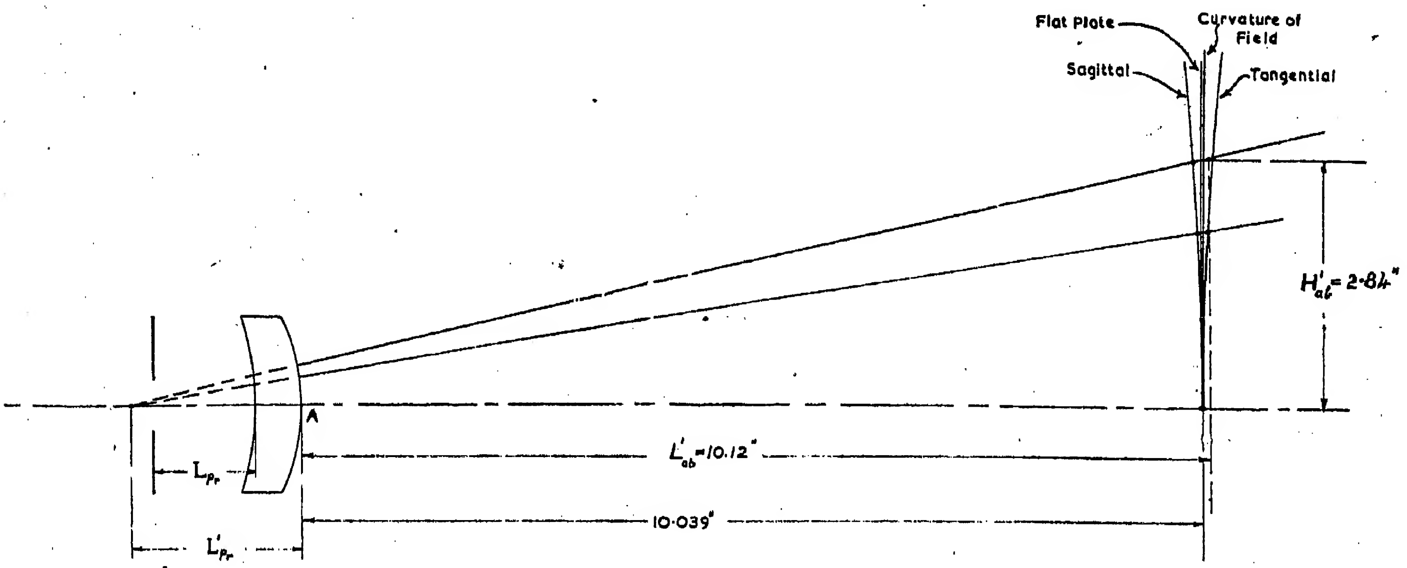


Fig. 35.

It is important to know the diameters of the discs of least confusion on the axis and at the particular inclinations taken, in order to judge the quality of the definition. These may be obtained from :—

Diameter of D.L.C. (axial rays) $= \frac{1}{2}\, LA'_M .\ \tan U'_M$
Diameter of D.L.C. (oblique rays) $=$ Astigmatism $\times \tan U'_M$.

Inserting the various values obtained from the computing work, we find :—

Diameter of D.L.C. (axial) $= 0{\cdot}1098 \times \tan 1°\text{–}49'\text{–}2'' = 0{\cdot}0035''$.
Diameter of D.L.C. (oblique 13°) $= 0{\cdot}2256 \times \tan 1°\text{–}49'\text{–}2'' = 0{\cdot}0072''$
Diameter of D.L.C. (oblique 9°) $= 0{\cdot}1150 \times \tan 1°\text{–}49'\text{–}2'' = 0{\cdot}0037''$

In order to interpret these values in terms of quality of definition on the photographic plate, the following tolerances may be employed as a useful standard :—

For extremely sharp definition, diameter of D.L.C. $= 0{\cdot}001''$
For good definition, diameter of D.L.C. $= 0{\cdot}004''$
For (so-called) soft definition, diameter of D.L.C. $= 0{\cdot}010''$.

Thus we see, that our lens would give reasonably good definition over the whole of the plate whose diagonal dimension is six inches.

It only remains now to obtain the amount of the distortion from the trigonometrically traced rays; this being determined from :—

$$\text{Distortion}' = -\left\{\frac{N}{N'} .\ f' .\ \tan U_{pr}\right\} - \left\{(L'_{pr} - l') \tan U'_{pr}\right\}$$

where N and N' are the refractive indices of the media in the object space

and image space respectively; so that in the case of a photographic lens, both N and N' being air the value $\frac{N}{N'}$ is unity. And f' is the equivalent focal length of the lens.

Referring to the calculations and inserting the appropriate values :—

$$\text{Distortion}' = \left\{-9\cdot6347\ (\tan -16^\circ\text{–}42'\text{–}0'')\right\} - \left\{-1\cdot9149 - 10\cdot2041\ (\tan -13^\circ\text{–}15''\text{–}20'')\right\}$$

$$\begin{array}{rlrl} \log -9\cdot6347 = & 0\cdot98384n & \log -12\cdot1190 = & 1\cdot08347n \\ +\log\tan -16^\circ\text{–}42'\text{–}0'' = & 9\cdot47714n & +\log\tan -13^\circ\text{–}15'\text{–}20'' = & 9\cdot37212n \\ \log \text{ 1st term} = & 0\cdot46098 & \log \text{ 2nd term} = & 0\cdot45559 \\ \text{1st term} = & 2\cdot8905'' & \text{2nd term} = & 2\cdot8549'' \end{array}$$

$$\therefore \quad \text{Distortion} = 2\cdot8905 - 2\cdot8549 = +\,0\cdot0356''.$$

On looking at the distortion formula it will be seen that the first term gives the *ideal* image height (i.e., when no distortion is present) and the second term gives the *actual* image height. These values are indicated on the drawing in Fig. 30, and in order to determine the type of the distortion it is necessary to know the height h_2 with respect to h_1. As the distortion varies as $(H'_k)^3$,

$$\text{the height } h_2 = \left(\frac{\text{OB}}{\text{OA}}\right)^3 \times 0\cdot0356 = \left(\frac{2\cdot044}{2\cdot8905}\right)^3 \times 0\cdot0356 = 0\cdot0126''.$$

(OB $= 2\cdot8905 \times \cos 45^\circ = 2\cdot044''$.)

And $h_1 = 0\cdot0356 \times \cos 45^\circ = 0\cdot0252$, hence the distortion is of the "barrel" type.

Sometimes the distortion may be expressed as the linear distance $(h_1 - h_2)$; that is, the departure from a true straight line, in which case the distortion value would be $0\cdot0126''$.

As a Distortion Tolerance, it may be stated that in a length of 4″ it is just possible to discern a curve whose departure $(h_1 - h_2)$ from a straight line is $0\cdot02''$ at the centre point.

Summarising the various aberrations from all the calculations, we find :—

$$\text{Spherical Aberration} = +\,0\cdot2196''$$
$$\text{Coma}'_T = +\,0\cdot0001''$$
$$\text{Coma}'_S = +\,0\cdot00003''$$

Best diaphragm position $= 1\cdot140''$ from the front surface of the lens.

$$\text{Astigmatism} = -\,0\cdot2256'' \text{ at } 13^\circ$$
$$\text{Astigmatism} = -\,0\cdot1150'' \text{ at } 9^\circ$$

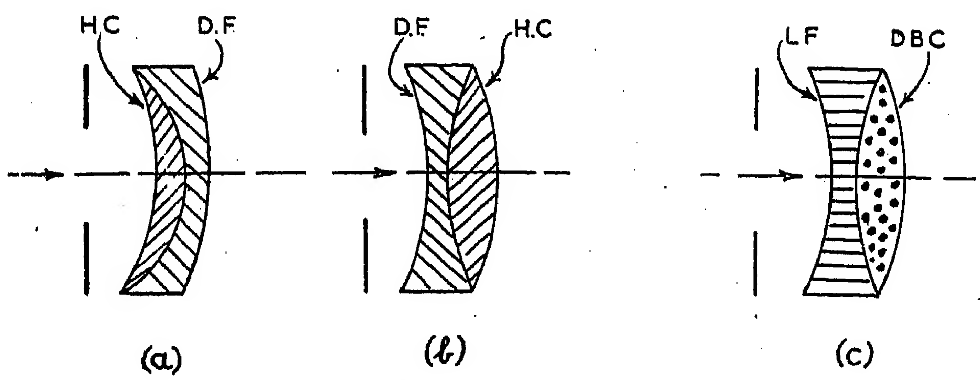
Best shape of lens for securing a flat field $= \left\{ \begin{array}{l} r_1 = -3 \cdot 333'' \\ r_2 = -2 \cdot 304'' \end{array} \right.$

Distortion $= 0 \cdot 0126''$ in a length of four inches at a distance of two inches from the centre of the plate.

Diameter of "discs of least confusion" $\left\{ \begin{array}{l} \text{(axial)} = 0 \cdot 0035'' \\ \text{(oblique } 9^\circ) = 0 \cdot 0037'' \\ \text{(oblique } 13^\circ) = 0 \cdot 0072'' \end{array} \right.$

Achromatized Meniscus Lenses

Two forms of achromatized meniscus lens are shown in Fig. 36 (a) and (b) in which Hard Crown and Dense Flint glasses were used ; and a third type



"OLD GLASS" ACHROMATS "NEW GLASS" ACHROMAT

Fig. 36.

(c) in which a Light Flint and Dense Barium Crown were employed. In these types it can be shown that it is necessary to have a fairly large amount of spherical aberration present in order to eliminate the coma (a distinctly worse aberration as far as photographic lenses are concerned). With type (c) it is possible to secure a greater reduction in astigmatism than with the other two.

When it is required to fulfil the Petzval and achromatism condition simultaneously, as for example in these particular cases, we have :—

$$\frac{1}{r_{ptz}} = \left(\frac{N_a - 1}{N_a}\right) R_a + \left(\frac{N_b - 1}{N_b}\right) R_b$$

where the suffixes a and b refer to the first and second lenses of the doublet.

But $R_a = \dfrac{1}{f \cdot \delta N_a (V_a - V_b)}$ and $R_b = -\dfrac{1}{f \cdot \delta N_b (V_a - V_b)}$

so that $\dfrac{1}{r_{ptz}} = \dfrac{1}{f\,(V_a - V_b)} \cdot \left(\dfrac{V_a}{N_a} - \dfrac{V_b}{N_b}\right)$

From this it will be seen that in order to secure a flat field the requirements are that the V and N values of the two glasses should rise and fall together.

A list of "old" glasses (see below) shows that as the N values rise the V values fall. This is contrary to the Petzval theorem, and a list of "new" glasses (e.g., the barium crown series) illustrates that the drop in V values with a rise in N is very much less.

	"Old" Glasses				*"New" Glasses*		
	N	V	V/N		N	V	V/N
(1) H.C.	1·5175	60·5	39·9	(1) L.B.C.	1·5407	59·4	38·6
(2) L.F.	1·5427	47·5	30·8	(2) M.B.C.	1·5744	57·7	36·6
(3) D.F.	1·6501	33·6	20·4	(3) D.B.C.	1·6140	56·9	35·2

A numerical example will reveal the difference in the Petzval curvature given when combining glasses Nos. 1 and 3 from the left-hand column; and No. 2 from the left column combined with No. 3 from the right-hand column. A focal length of ten inches will be assumed.

$$\text{(i)} \quad \frac{1}{r_{ptz}} = \frac{\left(\dfrac{V_a}{N_a} - \dfrac{V_b}{N_b}\right)}{f\,(V_a - V_b)} = \frac{19\cdot5}{10 \times 26\cdot9} = 0\cdot0725 \quad \therefore \quad r_{ptz} = 13\cdot8''$$

$$\text{(ii)} \quad \frac{1}{r_{ptz}} = \frac{\left(\dfrac{V_a}{N_a} - \dfrac{V_b}{N_b}\right)}{f\,(V_a - V_b)} = \frac{4\cdot4}{10 \times 9\cdot4} = 0\cdot0468 \quad \therefore \quad r_{ptz} = 21\cdot4''$$

It will be found interesting to experiment with different pairs of glasses from a glass catalogue on the lines set out above, to see how the radius of curvature of the Petzval surface may be still further increased.

The preceding section gave the general outline of the procedure for the design of the meniscus type of photographic lens, but one glass only was used. The principles involved in that section (with some modifications) will now be applied to the design of two of the forms of achromatized meniscus lenses described above. As examples we will take (i) the "old glass" achromat and (ii) the "new glass" achromat; for by keeping similar requirements as to focal length, aperture and angular field for both types a comparison in relative performance can be made and will prove distinctly instructive. Moreover, the advantage to be gained when using the barium crown series of glasses will be clearly illustrated.

We will ask for the following specification for both types of lenses:—

Equivalent focal length to be ten inches.

Full aperture to be 0·6 inches (or larger if the aberrations will permit).

To cover a plate whose distance across its diagonal is six inches (i.e., $H'_k = 3''$) or a semi-angular field of 16°–42′–0″.

"Old glass" Achromat

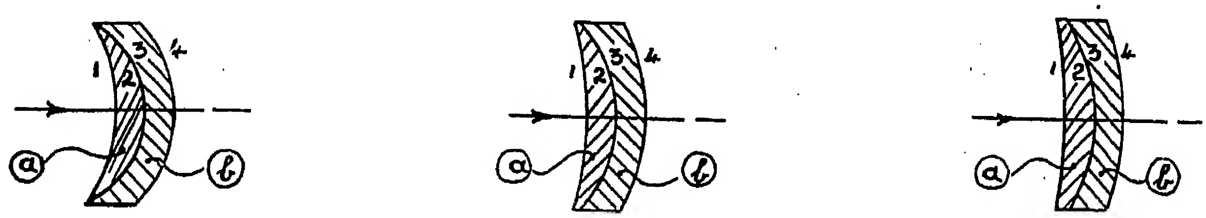
Dealing therefore with the "old glass" achromat first, we will stipulate that the lens system be of the Grubb form with crown lens in front and flint lens behind (see Fig. 36 (*a*)). The two glasses which were frequently employed in this type of lens (and which will be used in this example) are given below :—

Type of Glass	N_D	$\delta N = (N_{G'} - N_D)$	$N_{G'}$	V
Hard Crown	1·5186	0·0110	1·5296	60·3
Dense Flint	1·6041	0·0211	1·6252	37·8

The first part of the procedure consists in determining the total curvatures $\mathcal{R}_a$ and $\mathcal{R}_b$ of each component of the system which will give the desired focal length and freedom from chromatic aberration for the D and G' lines of the visible spectrum, from :—

$$\mathcal{R}_a = \frac{1}{f'(V_a - V_b)\,\delta N_a} \quad \text{and} \quad \mathcal{R}_b = \frac{1}{f'(V_b - V_a)\,\delta N_b}.$$

It should be pointed out that owing to the meniscus shape of the lens combination and its considerable thickness it is necessary to substitute for f' a value considerably smaller than the nominal ten inches (originally specified) in



$R_1 = -0·800$	$r_1 = -1·250''$	$R_1 = -0·400$	$r_1 = -2·500''$	$R_1 = -0·200$	$r_1 = -5·000''$
$R_2 = -1·341$	$r_2 = -0·746''$	$R_2 = -0·941$	$r_2 = -1·062''$	$R_2 = -0·741$	$r_2 = -1·349''$
$R_3 = -1·341$	$r_3 = -0·746''$	$R_3 = -0·941$	$r_3 = -1·062''$	$R_3 = -0·741$	$r_3 = -1·349''$
$R_4 = -1·060$	$r_4 = -0·943''$	$R_4 = -0·660$	$r_4 = -1·514''$	$R_4 = -0·466$	$r_4 = -2·172''$

Fig. 37.

order to get the correct back focal length. Seven and a half inches for f' will be found suitable to give the correct power of each of the components ; thus

$$\mathcal{R}_a = \frac{1}{7·5 \times 22·5 \times 0·0110} = +0·5411$$

and

$$\mathcal{R}_b = \frac{1}{7·5 \times -22·5 \times 0·0211} = -0·2807.$$

With these values, we may now "bend" the components into various shapes and determine the corresponding curvatures of the surfaces from $\mathcal{R}_a = R_1 - R_2$ and $\mathcal{R}_b = R_3 - R_4$. Assuming that the combination is to be a cemented one and that for example we put R_1 equal (in turn) to $-0·80$,

$-0 \cdot 40$ and $-0 \cdot 20$, we find the following shapes or bendings of the lens (see Fig. 37).

These shapes are then drawn to scale from which the axial thicknesses may be obtained; $0 \cdot 20$ inches will be found suitable for both the crown and flint lenses.

Taking, therefore, one of the shapes (say No. 1) we trace an incident ray at $\sqrt{0 \cdot 5}$ of the semi-aperture (namely, $\sqrt{0 \cdot 5} \times 0 \cdot 300 = 0 \cdot 2121''$) through the first and second surfaces using the refractive indices for D and G' light of each glass. The correct last radius for complete achromatism is then obtained from $\dfrac{1}{r} = \dfrac{(L_D - L_{G'})\, N_D}{L_D \,.\, L_{G'}\,(N_{G'} - N_D)} + \dfrac{1}{L_{G'}}$; this will be found to be $-1 \cdot 076''$. The D and G' rays are then continued on through the last surface, when the chromatic aberration will be found to be $+0 \cdot 015''$. The permissible tolerance obtained from $\pm \dfrac{0 \cdot 5\, \lambda}{N' \,.\, \sin^2 U'_{0 \cdot 7071}}$ is $\pm 0 \cdot 023''$.

Having corrected the achromatism, a marginal and paraxial ray (with $Y = 0 \cdot 300 \doteq y$) is then traced in G'-light; this gives $l'_k = 11 \cdot 926$, $L'_k = 10 \cdot 757$, $u'_k = 0 \cdot 02879$, and $U'_k = 1^\circ\text{–}47'\text{–}10''$.

At this stage we may now revert to the analytical formulæ (see page 79) and determine the SC', the CC', the AC', the PC' and the DC' values for this particular shape of lens. The arrangement of these calculations has already been shown on pages 80–85 but must now be carried out in three columns on account of the additional surface with this lens system.

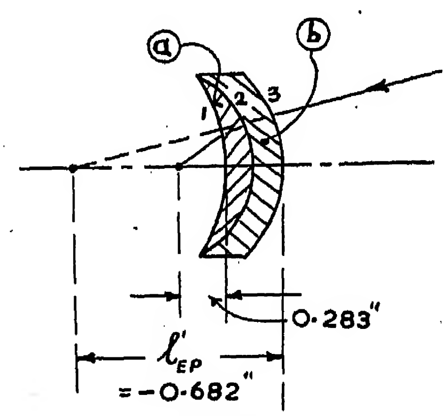


Fig. 38.

For the calculation of SC' the values from the paraxial ray trace are employed; but for the CC' calculation we require the i'_{pr} values at each surface when the diaphragm is in its correct position for the elimination of coma; this position is not yet known. Instead of making three trial stop distances with their accompanying three principal-ray tracings (as we did earlier in the case of the single meniscus lens) a considerable saving of time may be effected by using the following formula:—

$$l'_{EP} = l'_k - \frac{(l'_k - L'_k)(1 - OSC')}{(1 - OSC) - \left(\dfrac{\sin U_1}{u_1} \cdot \dfrac{u'_k}{\sin U'_k}\right)}$$

Putting the values from the marginal and paraxial ray-trace into this equation and OSC' equal to zero, this gives the distance l'_{EP} of the exit pupil from the last surface of the lens (see Fig. 38) for the elimination of coma;

l'_{EP} in this case is $-0\cdot682''$. Now, by definition the exit pupil is the virtual image of the diaphragm as seen through the lens from the rear, so that in order to find the *real* position of the diaphragm or stop we must trace a ray from *right to left* using the value of l'_{EP} as the object distance and a value of $u_{pr} = -0\cdot281180$ corresponding to the angular inclination of the emergent ray which gives the required image height H'_k. This ray-trace (shown in Calculation No. 27) serves a double purpose, for besides giving the position of the diaphragm it provides the values for i'_{pr} at each surface for the calculation of the CC′.

CALCULATION NO. 27

Surface	(3)	(2)	(1)
l $-r$	−0·682 +1·076	−0·5938 +0·746	−0·3863 +1·250
$(l-r)$	+0·394	+0·1522	+0·8637
$\log u$ $+\log(l-r)$	9·44898*n* 9·59550	9·38305*n* 9·18242	9·38855*n* 9·93636
$\log(l-r)u$ $-\log r$	9·04448*n* 0·03181*n*	8·56547*n* 9·87274*n*	9·32491*n* 0·09691*n*
$\log i$ $+\log\left(\frac{N}{N'}\right)$	9·01267 9·78909	8·69273 0·02633	9·22800 0·18458
$\log i'$ $+\log r$	8·80176 0·03181*n*	8·71906 9·87274*n*	9·41258 0·09691*n*
$\log r.i'$ $-\log u$	8·83357*n* 9·38305*n*	8·59180*n* 9·38855*n*	9·50949*n* 9·52398*n*
$\log(l'-r)$	9·45052	9·20325	9·98551
u $+i$	−0·281180 +0·102960	−0·241572 +0·049287	−0·244652 +0·169044
$u+i$ $-i'$	−0·178220 −0·063352	−0·192285 −0·052367	−0·075608 −0·258571
u'	−0·241572	−0·244652	−0·334179
$l'-r$ $+r$	+0·2822 −1·076	+0·1597 −0·746	+0·9672 −1·250
l' $-d'$	−0·7938 +0·200	−0·5863 +0·20	−0·2828
new l	−0·5938	−0·3863	

N.B.—In tracing *right to left*, the axial thickness d' must be added, not subtracted.

"Old Glass" Achromat.

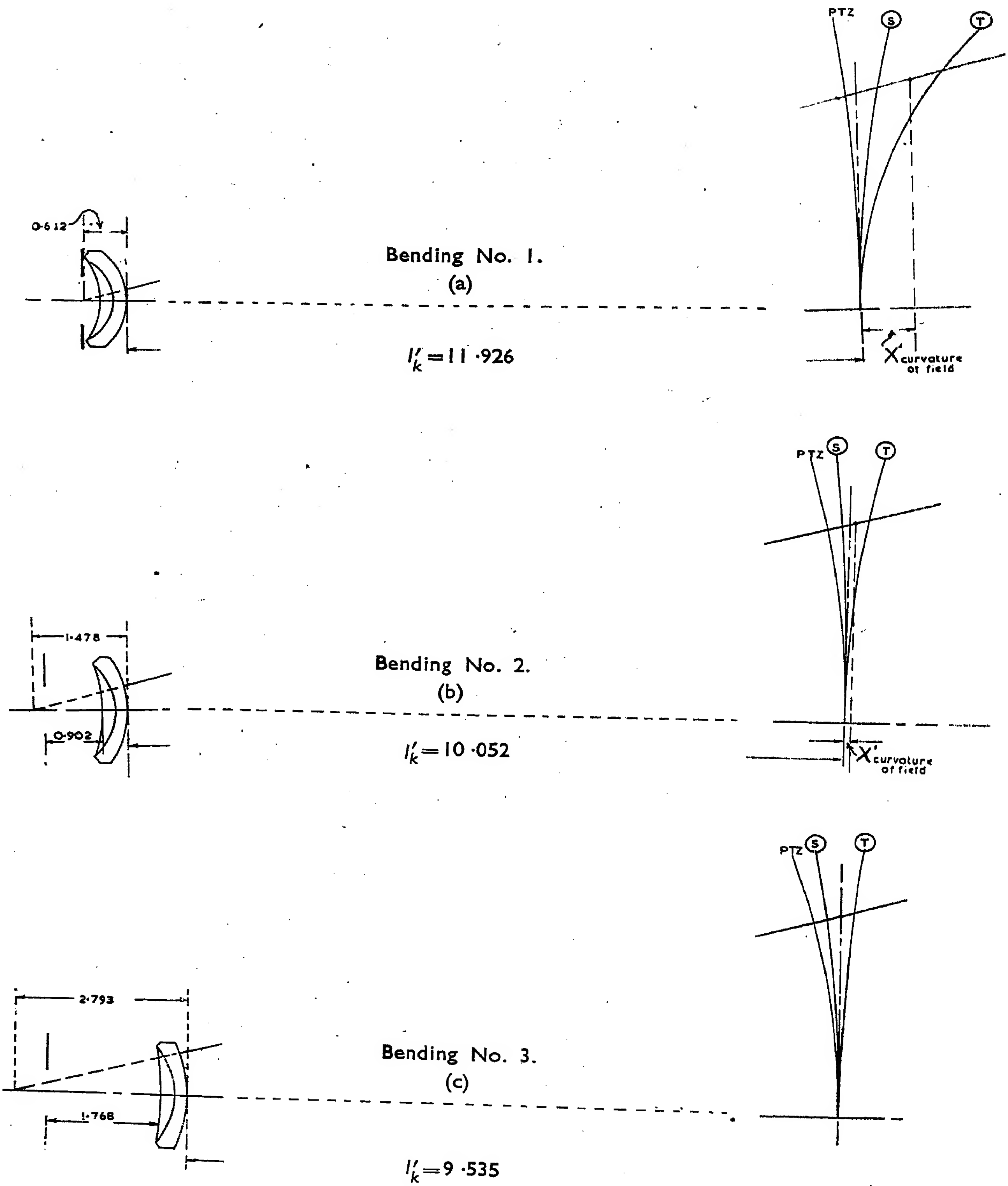


Fig. 39.

Thus, the real position of the diaphragm for the elimination of coma is $-0 \cdot 283$ inches from the first surface of the lens ; it is advisable, however, to check this, and the CC' equation may be used for this purpose. In carrying out this calculation it should be noted that the i'_{pr} values to be used are in reality the i_{pr} values given in Calculation No. 27 because this ray-tracing is *right to left.* The formula on page 79 gives a total CC' of $+0 \cdot 0007$ which may be considered as quite small enough at this stage for the elimination of coma. The AC' value will be found to be $-0 \cdot 4878$, whilst the $PC' = +0 \cdot 1824$ (i.e., $r_{ptz} = 34 \cdot 53$) and $DC' = +0 \cdot 0543$.

The same procedure must now be carried out for the two other lens shapes or bendings and the following table shows the various values which will be obtained.

Shape No.	*Corrected last radius*	*Chromatic Aberration*	*Spherical Aberration*	*Distance of exit pupil from surface r_4*	*Real distance of diaphragm from surface r_1*
1	−1·076	+0·015	+1·169	−0·682	−0·283
2	−1·623	+0·002	+0·326	−1·486	−0·902
3	−2·261	+0·0008	+0·122	−2·793	−1·768

Shape No.	*SC′*	*CC′*	*AC′*	*Astigmatism (actual)*	*PC′*	r_{Ptz}	*DC′*
1	+1·179	+0·0007	−0·4878	−0·9756	+0·1824	34·53	+0·0543
2	+0·327	0·0000	−0·2036	−0·4072	+0·3280	16·21	+0·0481
3	+0·124	+0·0001	−0·2123	−0·4246	+0·4349	14·03	+0·0715

The shape of the astigmatic fields are then drawn out to scale for each of the lens shapes (see Fig. 39) as described in detail in the previous section (pp. 86–87). At this stage it is advisable to plot the values for the spherical aberration and the astigmatism against the shape of the lens (e.g., against the curvature R_1 of the first surface) see Fig. 40. If then we measure from our scale drawings already made (Fig. 39) the horizontal distance of the mid-point of the sagittal and tangential foci from the perpendicular passing through the paraxial focus (i.e., $X'_{(curvature\ of\ field)}$) we shall get the position of the D.L.C. (oblique rays) with respect to the latter. If this is done for the three shapes of the lens, points will be obtained for the plotting of an additional curve on the same graph ; where this curve crosses the axis, the shape of the lens will be obtained which secures a zero value for the true curvature of field (i.e., the field will be flat). In this case, from the graph $R_1 = -0 \cdot 200$ and hence the best shape of the lens will be that already chosen for Bending No. 3 which has radii $r_1 = -5 \cdot 000''$; $r_2 \equiv r_3 = -1 \cdot 349''$; and $r_4 = -2 \cdot 172''$ (not corrected).

N.B.—The scale drawing of Fig. 39c would have been sufficient in itself to tell us the best shape of lens to employ without plotting the X' (*curvature of field*) curve. It seldom happens, however, that an arbitrarily chosen bending gives the correct solution to the problem; and therefore it is advisable to find this systematically by plotting this curve.

The above specification must now be tested trigonometrically throughout. First, the chromatic aberration is corrected by tracing D and G' rays at $\sqrt{0 \cdot 5}$ of the semi-aperture with adjustment of the last radius to give complete achromatism. Secondly, marginal and paraxial rays are traced in G'-light in order to determine the spherical aberration. The position of the exit pupil is then calculated, and a *right-to-left* ray tracing made in order to secure the *real* position of the diaphragm for the elimination of coma. This must be checked by tracing three oblique rays a, pr, and b through the

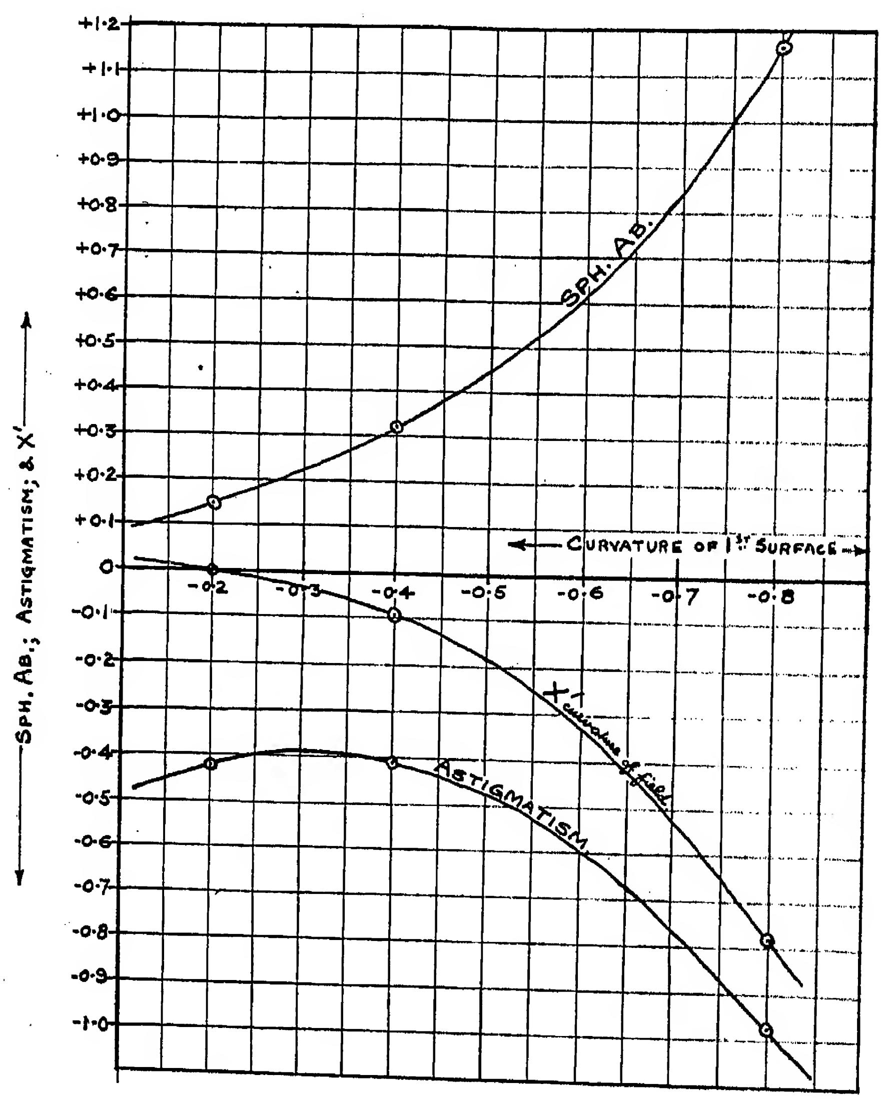


Fig. 40.

diaphragm at an inclination of -22 degrees by the method described on page 89. This will give the coma$'_T$, and the coma$'_S$ will be one third of the former value, both of which should be very small or negligible (it is advisable

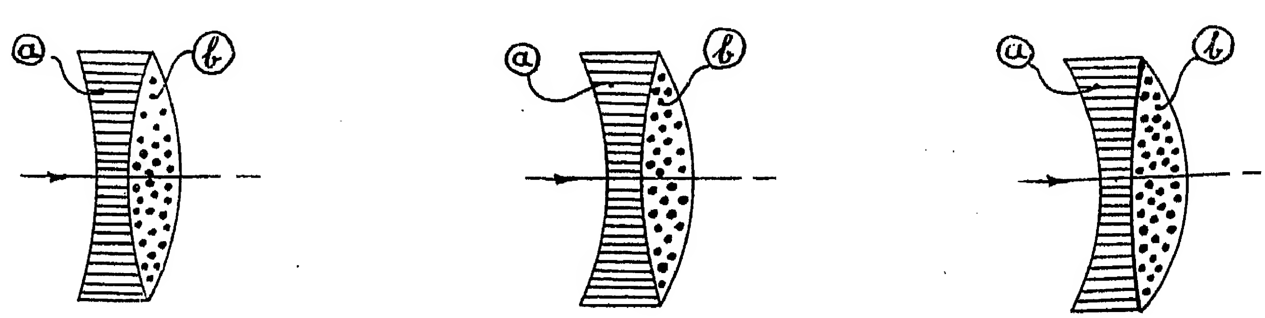
to have the coma patch size rather less than 0·0025 of an inch). Utilizing the values obtained in the principle ray oblique tracing, the astigmatism may be calculated as already shown on page 90. The position and size of the discs of least confusion (D.L.C.) are then worked as indicated on page 93 and the final solution drawn out to scale. It will be found that this bending of the lens gives practically a flat field. Lastly the distortion is calculated.

The following are the numerical values given by the trigonometrical tests carried out on this " best solution " to the problem :—

Corrected last radius	$= -2{\cdot}261''$
Chromatic aberration ($L'_D - L'_G$)	$= +0{\cdot}0008''$
Spherical aberration ($l'_{G'} - L'_{G'}$)	$= +0{\cdot}1218''$
Distance of diaphragm from first surface of the lens (for elimination of coma)	$= -1{\cdot}768''$
Astigmatism (at $16\frac{1}{2}^\circ$ obliquity)	$= -0{\cdot}4820$
Distortion	$= +0{\cdot}0841$
Diameter of Discs of Least Confusion :	
axial	$= 0{\cdot}0020''$
$16\frac{1}{2}^\circ$ obliquity	$= 0{\cdot}0159''$

" New glass " Achromat

The advantage in using the barium crown glasses (or other glasses with a high N value combined with a high V value) has been explained very briefly on page 96 and some numerical examples illustrating the practical application of such glasses to the design of photographic lenses has been given in *Practical Optics* (B. K. Johnson), pp. 107–111. But full appreciation of the advantages to be gained can only be secured by the systematic design and computation of a specific lens system, and we cannot do better than carry out such a design having the same specified requirements as that used for the " old glass " achromat (given on page 96), and then compare the results obtained in each case.



$R_1 = -0{\cdot}3183$	$r_1 = -3{\cdot}142''$	$R_1 = -0{\cdot}4183$	$r_1 = -2{\cdot}390''$	$R_1 = -0{\cdot}5183$	$r_1 = -1{\cdot}930''$
$R_2 = +0{\cdot}3550$	$r_2 = +2{\cdot}817''$	$R_2 = +0{\cdot}2550$	$r_2 = +3{\cdot}922''$	$R_2 = +0{\cdot}1550$	$r_2 = +6{\cdot}452''$
$R_3 = +0{\cdot}3550$	$r_3 = +2{\cdot}817''$	$R_3 = +0{\cdot}2550$	$r_3 = +3{\cdot}922''$	$R_3 = +0{\cdot}1550$	$r_3 = +6{\cdot}452''$
$R_4 = -0{\cdot}4448$	$r_4 = -2{\cdot}248''$	$R_4 = -0{\cdot}5448$	$r_4 = -1{\cdot}835''$	$R_4 = -0{\cdot}6448$	$r_4 = -1{\cdot}551''$

Fig. 41.

"New Glass" Achromat.

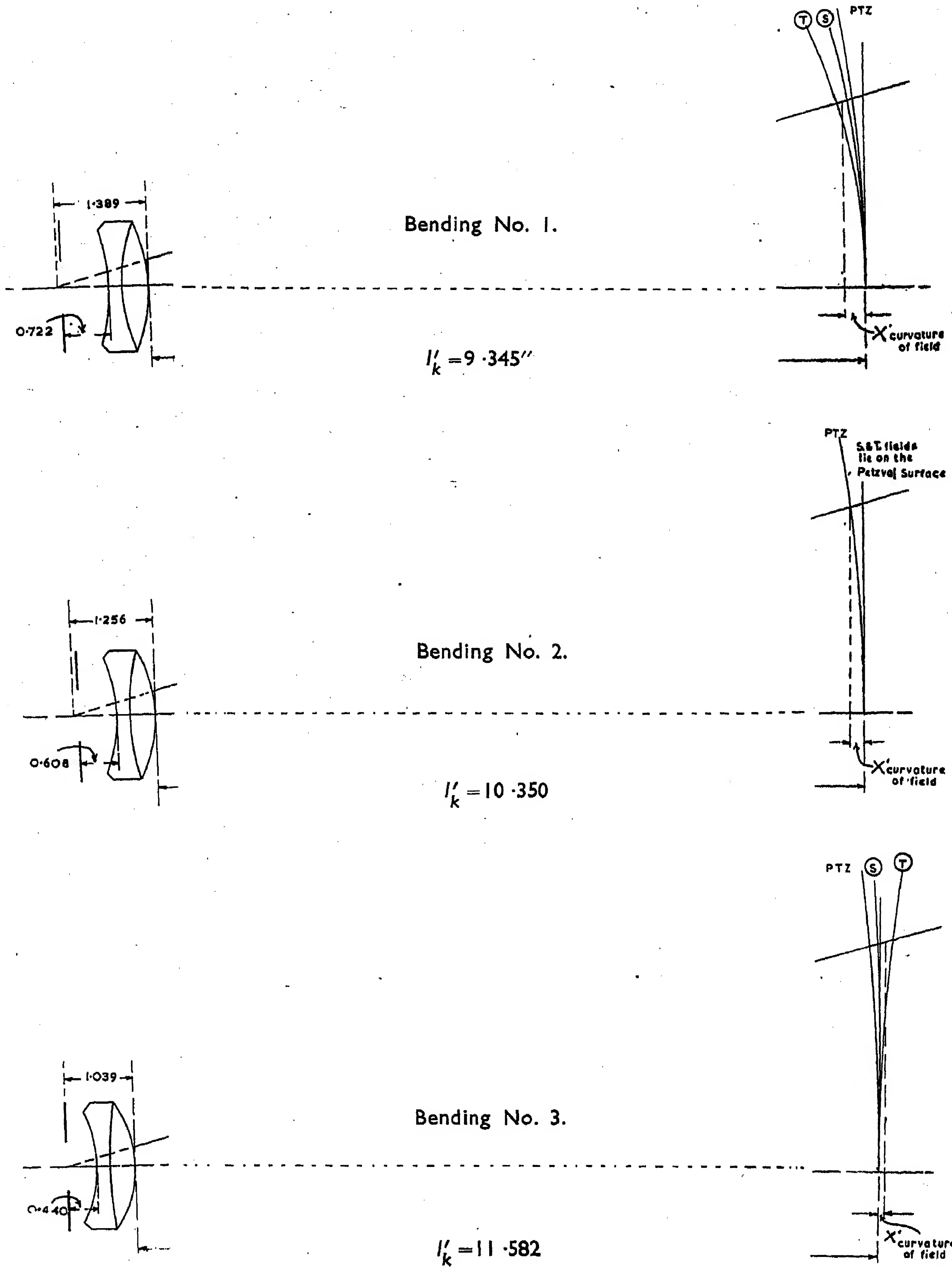


Fig. 42.

Conforming, therefore, with the procedure set out in the previous section we will commence by calculating the total curvature $\mathcal{R}_a$ and $\mathcal{R}_b$ of each component using the following glasses :—

Type	N_D	$\delta N = (N_{G'} - N_D)$	$N_{G'}$	V
Light Flint	1·5420	0·01572	1·5577	45·5
Dense Barium Crown ..	1·6059	0·01323	1·6191	59·0

Then $\mathcal{R}_a = \dfrac{1}{f'(V_a - V_b)\,\delta N_a} = \dfrac{1}{7 \times -13{\cdot}5 \times 0{\cdot}01572} = -0{\cdot}6733.$

and $\mathcal{R}_b = \dfrac{1}{f'(V_b - V_a)\,\delta N_b} = \dfrac{1}{7 \times 13{\cdot}5 \times 0{\cdot}01323} = +0{\cdot}7998.$

N.B.—The equivalent focal length (f') of the lens system must in this case be put as seven inches instead of the nominal ten inches specified, in order to give the required total curvature of each component.

Choosing now three shapes of the lens as trial bendings, we will in this case take the curvature of the contact surface R_2 equal to $+0{\cdot}355$, $+0{\cdot}255$ and $+0{\cdot}155$ in turn, and work out the corresponding radii of the surfaces utilizing the above values of $\mathcal{R}_a$ and $\mathcal{R}_b$, from $\mathcal{R}_a = R_1 - R_2$ and $\mathcal{R}_b = R_3 - R_4$. The shapes and radii are shown in Fig. 41, and from scale drawings suitable axial thicknesses will be 0·200″ and 0·400″ for lens (*a*) and lens (*b*) respectively.

The ray tracing with incident parallel light ($Y = 0{\cdot}2121$) for D and G′ light may then be commenced and the last radius adjusted (as already explained) to correct the chromatic aberration. The spherical aberration is also determined trigonometrically ; and from the latter calculation, values are substituted in the formula for giving the position of the exit pupil at which the lens will be free from coma. As before a *right-to-left* ray tracing is then made to find the real position of the diaphragm. Up to this stage the following values will be obtained :—

Shape No.	*Corrected last radius*	*Chromatic Aberration*	*Spherical Aberration*	*Distance of exit pupil from surface* r_4	*Real distance of diaphragm from surface* r_1
1	−2·458″	+0·0033″	+0·1933″	−1·389″	−0·722″
2	−2·084″	+0·0029″	+0·2788″	−1·256″	−0·608″
3	−1·827″(5)	+0·0013″	+0·4064″	−1·039″	−0·440″

The various values for the SC', CC', AC', PC' and DC' can then be determined by means of the analytical formulæ (given on page 79) already

described. The numerical quantities for these aberrations will be found to be :—

Shape No.	SC'	CC'	AC'	*Astigmatism (actual)*	PC'	r_{PTZ}	DC'
1	+0·1919	0·0000	+0·0850	+0·1700	0·1935	23·63	0·0647
2	+0·2797	+0·0002	−0·0175	−0·0350	0·2125	25·17	0·0863
3	+0·4021	+0·0001	−0·1341	−0·2682	0·1729	36·50	0·0649

The shape of the astigmatic fields, the Petzval curvature, the lens and its diaphragm position, are all drawn out to scale (as before) for each of the three "bendings" (see Fig. 42). The spherical aberration and the astigmatism values from the foregoing tables are then plotted against the shape of the lens (in this case the curvature R_2 of the contact surface) as shown in Fig. 43.

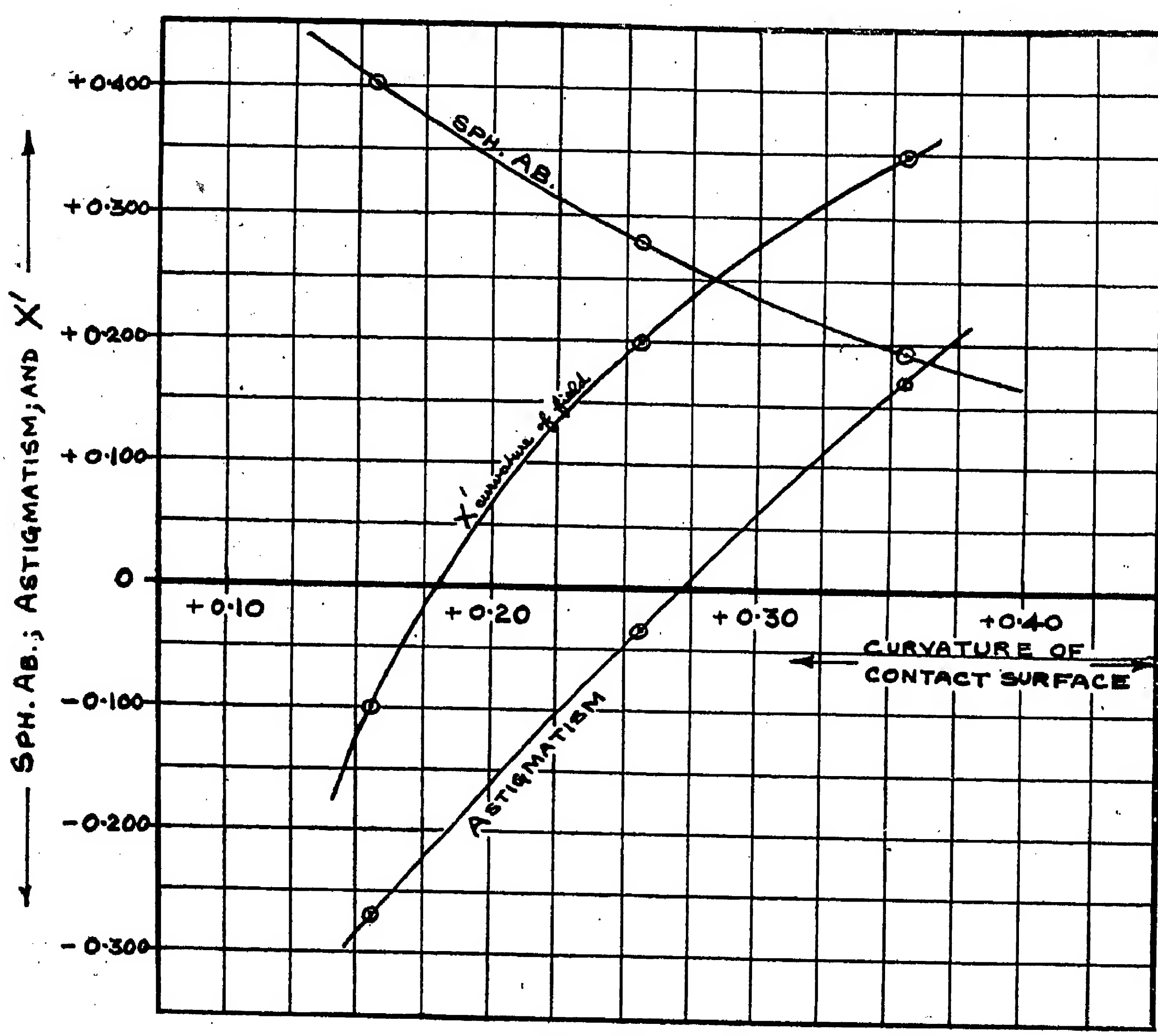


Fig. 43.

The distance of the D.L.C. (for the oblique rays) from the paraxial focus, namely $X'_{(curvature\ of\ field)}$, is also plotted for the three bendings; where this

curve cuts the abscissa (i.e., at $R_2 = +\,0 \cdot 180$) this will give the shape of the lens which will provide a flat field. The corresponding radii for this solution are therefore :—

$$r_1 = -\,2 \cdot 027'' ; \quad r_2 \equiv r_3 = +\,5 \cdot 556'' ; \quad \text{and } r_4 = -\,1 \cdot 613'' \text{(uncorrected)}$$

This must be tested trigonometrically throughout.

The final results given by this specification are as follows :—

Corrected last radius	$= -\,1 \cdot 880''$
Chromatic aberration	$= +\,0 \cdot 0010$
Spherical aberration	$= +\,0 \cdot 3657$
Distance of diaphragm from first surface of lens (for elimination of coma)	$= -\,0 \cdot 4736$
Astigmatism (at 18° obliquity)	$= -\,0 \cdot 2644''$
Distortion	$= +\,0 \cdot 1107''$
Diameter of Discs of Least Confusion :	
axial	$= \;0 \cdot 0056''$
at 18° obliquity	$= \;0 \cdot 0080''$

Observations on the Two Sets of Results

One of the more interesting points which reveals itself from a comparison of the two sets of results is the fact that with the " old glass " achromat the sagittal astigmatic surface is (in each of the three bendings) *nearer* the lens than the tangential field, whereas with the " new glass " achromat it is possible to change the fields over as illustrated by the results of shapes No. 1 and No. 3 of this latter type of lens shown in Fig. 42.

As an outcome of this fact it will be obvious that one could, by choosing a suitable bending of the lens, make the sagittal and tangential fields coincide which would therefore give complete freedom from astigmatism ; indeed this is the actual case depicted in shape No. 2 of the " new glass " achromat and shown in Fig. 42.

This coincidence of the sagittal and tangential fields implies also that they coincide with the Petzval curvature surface, but the latter has unfortunately a radius of $25 \cdot 17''$ (drawn to scale in Fig. 42, case No. 2) and consequently the image field (although having small discs of least confusion) would not lie on a flat photographic plate. Thus, it is necessary to introduce some astigmatism in order to swing the fields back a little from the lens and so produce a true curvature of field which shall be zero ; fortunately it is possible (by means of the so-called " new " glasses) not to introduce a large amount of astigmatism in securing the condition for a flat field. For example, the astigmatism at the extreme angular part of the field for the best shape of this lens is only $-\,0 \cdot 264$, whereas with the " old glass " achromat (best solution) at approximately the same angular field the astigmatism is $-\,0 \cdot 482''$.

Further, by comparing the graphs of Fig. 40 with those of Fig. 43 it will be noticed that with the " old glass " achromat the astigmatism never reaches a zero value, but with the "new glass" achromat both positive, negative and zero amounts of astigmatism can be secured. The solution conditional with a flat field for each form of lens indicates, however, that the spherical aberration is considerably greater with the "new glass" achromat than with the " old glass " type of lens ; but that the astigmatism is less in the former type than in the latter. Thus the D.L.C. in the centre of the plate would be larger in the case of the " new glass " achromat than with the " old ", and therefore the definition poorer ; whereas at the edge of the field (on account of its less astigmatism) the definition would be better than with the " old glass " achromat.

The above graphed results are obtained with the pairs of glasses we chose quite arbitrarily, but even so the general tendency in behaviour of the aberrations can be visualised. It becomes evident from these graphs that the best compromise in quality of definition will be secured by retaining the " new glass " form of lens system, but by experimenting with pairs of glasses which will move the spherical aberration curve (of Fig. 43) towards the left. By doing this, both the spherical aberration and the astigmatism may be kept down to small amounts for the condition of a flat field.

This gives an excellent opportunity for pursuing a useful design for an achromatized meniscus photographic lens, for as has been stated before the numerical examples given here are only intended to suggest the general lines along which the designing proper may be carried out.

A minor point, but nevertheless not an unimportant one, is the fact that the position of the diaphragm for the elimination of coma is considerably nearer (in our example, approximately three times nearer) to the lens in the case of the " new glass " achromat than with the " old glass " type. This means that the lens unit as a whole is more compact and does not take up so much space ; it is also a useful point to bear in mind when designing an achromatized symmetrical type of lens or an anastigmat.

Complete Procedure

It may be found useful to summarize the various points which have to be attended to when carrying out the systematic design of an achromatized meniscus lens and to put them down in their correct order ; hence the following procedure :—

(1) Determine the total curvature of each component from

$$R = \frac{1}{f'\,(V_a - V_b)\,\delta N}$$

N.B.—Choose a suitable value for f' to give the correct specified equivalent focal length.

(2) Choose three shapes of lens which appear suitable. This may be done by changing either the curvature of the first surface or the contact surface.

(3) With the first shape of lens decided on, trace rays at $\sqrt{0\cdot 5}$ of the semi-aperture in D and G′- light and correct the last radius for achromatism, from :—

$$\text{corrected } \frac{1}{r} = \frac{(L_D - L_{G'})\,N_D}{L_D \,.\, L_{G'}\,(N_{G'} - N_D)} + \frac{1}{L_{G'}}$$

(4) Marginal and paraxial rays in G′-light are then traced through the lens system.

(5) Utilizing the values from the paraxial ray-trace, the spherical aberration contribution (SC') is calculated from the analytical formula.

(6) Also using the ray-tracing values of (4) determine the position of the virtual image of the diaphragm (i.e., the exit-pupil) for the elimination of coma, from

$$l'_{EP} = l'_k - \frac{(l'_k - L'_k)(1 - \text{OSC}')}{(1 - \text{OSC}') - \left(\dfrac{\sin U_1}{u_1} \cdot \dfrac{u'_k}{\sin U'_k}\right)}$$

(7) With this value of l'_{EP}, a ray is then traced through the lens from *right-to-left* with an angle of inclination corresponding to that which gives the required image height (H'_k) for the specified focal length. The final intersection length l_{pr} will give the *real* position of the stop from the front surface of the lens.

(8) This position of the diaphragm should now be checked by employing the analytical formula for the coma contribution (CC') which necessitates the use of the i'_{pr} values at each surface obtained from the oblique ray-tracing already carried out in (7).

(9) Providing the total coma is zero, or nearly so, the astigmatic contribution (AC') can then be determined from the analytical formula for this.

(10) Similarly, the Petzval contribution (PC').

(11) The shape of the Petzval, sagittal and tangential surfaces, and the true curvature of field, are then drawn out on paper to scale.

(12) When operations 3 to 11 have been carried out for each of the three shapes or " bendings " of the lens, a survey of the results may be made and a likely solution chosen which will give the flattest possible field.

(13) The spherical aberration and the astigmatism values are now plotted against the shape of the lens ; also the distance of the D.L.C. (for the oblique rays) from the paraxial focus. Where this latter curve crosses the axis this will give the best shape of the lens for securing a flat field.

(14) This solution should then be tested trigonometrically throughout, and the residual aberrations obtained.

(15) The position of the discs of least confusion for the axial and oblique rays are then calculated and the shape of the field drawn out to scale.

(16) The size of the discs of least confusion are also determined in order to assess the quality of definition.

Symmetrical Types

The arrangement of two similar lenses placed symmetrically about a central diaphragm and with object and image situated at equal distances on either side of the lens, leads automatically to the correction of distortion, coma, and transverse chromatic aberration in photographic objectives.

Although the correction of these aberrations is only true for unit magnification, they are greatly reduced even when object and image distances are not the same. No advantage is to be gained in axial chromatic aberration, spherical aberration, astigmatism and curvature of field, by employing the "symmetrical principle"; nevertheless the fact that the three named aberrations in the first paragraph can be minimized has led to a considerable use of this principle in the design of photographic lenses. Fig. 44 (*a*), (*b*), (*c*) and (*d*) show diagrammatically four types of symmetrical lens, namely a non-achromatized, an "old glass" achromatized, a "new glass" achromatized form, and a non-cemented doublet type respectively.

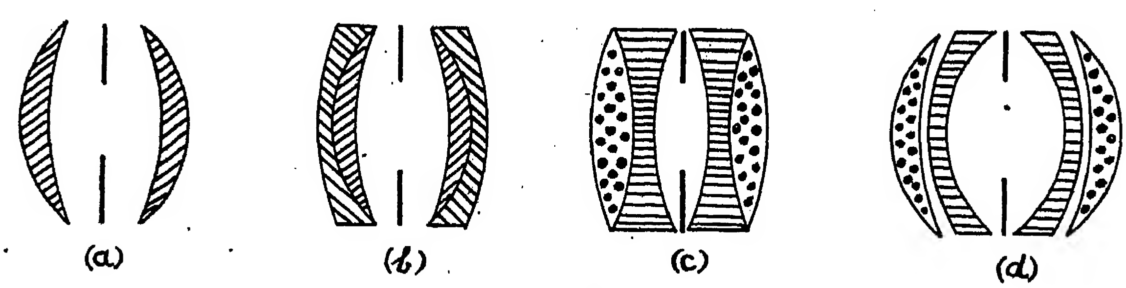


Fig. 44.

If we refer to the formulæ giving the effects of a shift in the position of the diaphragm on the aberrations (see Conrady's *Applied Optics*, p. 343) we find the one dealing with astigmatism, namely Ast* = Ast + 2Q.Coma + Q^2.Sph., enables a suitable choice of diaphragm-position to be made in order to reduce the astigmatism even though the spherical aberration is corrected, but at the same time it will be seen (from the formula) that it is necessary to have a certain amount of coma present in order to satisfy this condition. Therefore in designing a symmetrical type of lens, it is desirable to correct each component for axial chromatic and spherical aberration but to leave a fairly large amount of coma present.* By putting the components together

* In order to obtain large amounts of coma with zero spherical aberration, the choice of glasses is the dominant factor. The real requirement in this connection is a low V difference accompanied by a considerable N difference for the two glasses. To secure this aim, the earlier symmetrical lenses were made from a Light Flint and a Dense Flint for the two glasses of each component, whereas later an Extra Light Flint and a Dense Barium Crown were employed.

symmetrically about the diaphragm, the coma is then automatically removed or greatly reduced for the complete lens system, and by adjustment of the separation of the lenses the astigmatism and curvature of field may be controlled.

For the purpose of illustrating the design of a symmetrical-type lens, let us ask for the following specification :—

Equivalent Focal Length to be 10 inches.

Clear Aperture to be 1 inch or larger if the aberrations will permit.

To cover a half-plate ($6\frac{1}{2}'' \times 4\frac{3}{4}''$). This corresponds to an 8 inch diagonal (i.e., $H'_k = 4''$) or a semi-angular field of 22 degrees.

And the glasses as follows :—

	N_D	V	$N_{G'}$	$\delta N = (N_{G'} - N_D)$
Light Flint ..	1·56734	43·8	1·58426	0·01692
Dense Flint ..	1·61661	36·6	1·63887	0·02226

In commencing the design, we deal with the rear component first.

From elementary thin lens equations, the focal length of each component (assuming they were touching one another) would be twenty inches, but as they are to be of the meniscus shape and "thick" lenses it is found that f' must be 14·65″ instead of twenty.*

The total curvature of each lens comprising the rear component for achromatism will be obtained first of all, from

$$R_a = \frac{1}{f'(V_a - V_b)\,\delta N_a} = \frac{1}{14\cdot65 \times 7\cdot2 \times 0\cdot01692} = +\,0\cdot5603.$$

$$R_b = \frac{1}{f'(V_b - V_a)\,\delta N_b} = \frac{1}{14\cdot65 \times -7\cdot2 \times 0\cdot02226} = -\,0\cdot4259.$$

Although the rear component is in practice used in a slightly convergent beam, it will be found perfectly satisfactory in most cases to deal with the design on the assumption that the incident light is a parallel beam. We may now therefore find the extent of the spherical aberration and the coma for various shapes of this achromatized half of the complete lens system by means of the analytical G-sum formula (as already used on pages 20–21 in chapter II). The numerical work is shown in Calculation No. 28.

* If this adjustment was not made at this stage and the back focal length was in consequence longer than specified, the calculations could of course be carried out but the resulting aberrations would have to be changed in scale by the ratio of the specified B.F.L. to the determined B.F.L. The tolerances would, however, have to be re-calculated. It is probably better, therefore, to determine the right curvatures for R_a and R_b at the outset, as has been done above.

The spherical aberration contribution :—

$$LA' = y^2(l_b')^2[G_1\mathcal{R}_a^3 + G_2\mathcal{R}_a^2R_2 - G_3\mathcal{R}_a^2v_2' + G_4\mathcal{R}_aR_2^2 - G_5\mathcal{R}_av_2'R_2 + G_6\mathcal{R}_a(v_2')^2]$$
$$+ y^2(l_b')^2[G_1\mathcal{R}_b^3 - G_2\mathcal{R}_b^2R_3 + G_3\mathcal{R}_b^2v_3 + G_4\mathcal{R}_bR_3^2 - G_5\mathcal{R}_bv_3R_3 + G_6\mathcal{R}_b(v_3)^2]$$

and the coma contribution CC′ :—

$$CC' = H_k'.\mathrm{SA}^2[\tfrac{1}{4}G_5^a\mathcal{R}_aR_2 - G_7^a\mathcal{R}_av_2' + G_8^a\mathcal{R}_a^2 + \tfrac{1}{4}G_5^b\mathcal{R}_bR_3 - G_7^b\mathcal{R}_bv_3 - G_8^b\mathcal{R}_b^2]$$

Calculation No. 28

Rear Component

Front lens (light flint) :

$N_{G'} = 1\cdot58426$; $\mathcal{R}_a = +0\cdot5603$;

$(l_b')^2 = 318\cdot98$ $\quad v_2' = \dfrac{1}{l_2'} = (N_a - 1)\mathcal{R}_a \quad \log v_2' = 9\cdot5150$;

	$G_1\mathcal{R}_a^3$	$+G_2\mathcal{R}_a^2R_2$	$-G_3\mathcal{R}_a^2v_2'$	$+G_4\mathcal{R}_aR_2^2$	$-G_5\mathcal{R}_av_2'R_2$	$+G_6\mathcal{R}_a(v_2')^2$
log G + log $(l_b')^2$ + log $\mathcal{R}_a^n$ + log $(v_2')^n$	9·8652 2·5038 9·2452	0·0856 2·5038 9·4968	0·2255n 2·5038 9·4968 9·5150	9·8200 2·5038 9·7484	0·2802n 2·5038 9·7484 9·5150	0·0952 2·5038 9·7484 9·0300
log sum	1·6142	2·0862R_2	1·7411n	2·0722R_2^2	2·0474nR_2	1·3774
antilogs	41·134	121·96R_2	−55·094	118·08R_2^2	−111·53R_2	23·845

Sph. Ab. contribution from lens $a = y^2 \times (9\cdot885 + 10\cdot43R_2 + 118\cdot08R_2^2)$

Back lens (dense flint):

$N_{G'} = 1\cdot63887$; $\mathcal{R}_b = -0\cdot4259$; $\log v_3 = 9\cdot5150$;

$(l_b')^2 = 318\cdot98$.

	$G_1\mathcal{R}_b^3$	$-G_2\mathcal{R}_b^2R_3$	$+G_3\mathcal{R}_b^2v_3$	$+G_4\mathcal{R}_bR_3^2$	$-G_5\mathcal{R}_bv_3R_3$	$+G_6\mathcal{R}_bv_3^2$
log G + log $(l_b')^2$ + log $\mathcal{R}_b^n$ + log $(v_3)^n$	9·9335 2·5038 8·8879n	0·1356n 2·5038 9·2586	0·2765 2·5038 9·2586 9·5150	9·8508 2·5038 9·6293n	0·3133n 2·5038 9·6293n 9·5150	0·1297 2·5038 9·6293n 9·0300
log sum	1·3252n	1·8980nR_3	1·5539	1·9839nR_3^2	1·9614R_3	1·2928n
antilogs	−21·145	−79·068R_3	35·801	−96·361R_3^2	91·496R_3	−19·624

Sph. Ab. contribution from lens $b = y^2 \times (-4\cdot968 + 12\cdot428R_3 - 96\cdot361R_3^2)$

As the two lenses of the component are to be cemented, $R_2 = R_3$ and therefore the total spherical aberration of the rear doublet is :—

$$\begin{aligned} & 9\cdot885 + 10\cdot43\,R_2 + 118\cdot08R_2^2 \\ & -4\cdot968 + 12\cdot43\,R_2 - \ \ 96\cdot36R_2^2 \\ LA' = y^2 \Big\{ & \underline{\underline{4\cdot917 + 22\cdot86\,R_2 + \ \ 21\cdot72R_2^2}} \Big\} \end{aligned}$$

Substituting suitable values for R_2 we may now find the amount of the spherical aberration for various shapes of the lens ; the following table shows the values :—

Curvature of contact surface								
R_2 =	$-0\cdot20$	$-0\cdot30$	$-0\cdot40$	$-0\cdot50$	$-0\cdot60$	$-0\cdot70$	$-0\cdot80$	$-0\cdot90$
Sph. Ab. (LA') =	$+1\cdot214$	$+0\cdot014$	$-0\cdot752$	$-1\cdot083$	$-0\cdot980$	$-0\cdot442$	$+0\cdot530$	$+1\cdot936$
(For $y = 0\cdot5''$) LA' =	$+0\cdot303$	$+0\cdot003$	$-0\cdot188$	$-0\cdot271$	$-0\cdot245$	$-0\cdot111$	$+0\cdot132$	$+0\cdot484$

The last row of values can then be plotted against the curvature of the contact surface R_2 when the familiar parabola should be obtained (Fig. 45).

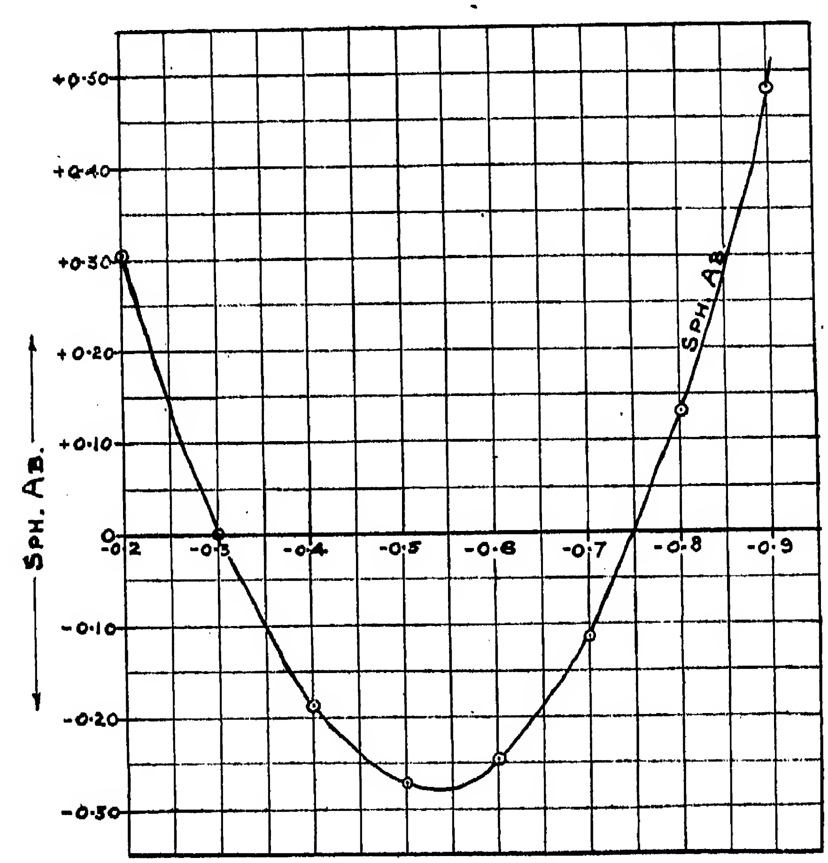


Fig. 45

At this stage it is not necessary to carry out a coma calculation for this half of the complete lens system, for the symmetrical nature of the latter will in itself reduce the coma to a small amount.

Referring to the spherical aberration curve of Fig. 45, the positions at which the parabola crosses the abscissa (i.e., zero spherical aberration) indicate values of $R_2 = -0 \cdot 300$ and $-0 \cdot 750$. These give two shapes of the lens having the following respective radii :—

$$\left.\begin{aligned} r_1 &= +3 \cdot 842'' \\ r_2 \equiv r_3 &= -3 \cdot 333'' \\ r_4 &= +7 \cdot 943'' \end{aligned}\right\} \quad \text{and} \quad \left\{\begin{aligned} r_1 &= -5 \cdot 271'' \\ r_2 \equiv r_3 &= -1 \cdot 333'' \\ r_4 &= -3 \cdot 085'' \end{aligned}\right.$$

these shapes should be drawn out to scale as shown diagrammatically in Fig. 46 (A) and Fig. 46 (B).

As the second solution (namely with the contact curvature $R_2 = -0 \cdot 750$) gives a more steeply curved meniscus form of lens, we will choose this as one half of the completely symmetrical lens system we are designing.

Proceeding with the design we must first test trigonometrically the chromatic and the spherical aberrations of the rear component. Incident parallel light can be used for this test (with $Y = \sqrt{0 \cdot 5} \times 0 \cdot 50$ for the chromatic test and $Y = y = 0 \cdot 50$ for the sph. ab. test) without committing any serious error, even though the rear component is used in a slightly convergent beam produced by the front component. In carrying out this test, the last radius is corrected (in the usual way) to give chromatic correction for D and G′ light ; and if the spherical aberration is found to exceed greatly the tolerance for the latter, the rear component must be "bent" as a whole in order to correct this. In point of fact, it is found necessary to change slightly the curvature of each surface of this lens by an amount equal to $-0 \cdot 028$; this gives a new shape with $r_1 = -4 \cdot 426''$; $r_2 \equiv r_3 = -1 \cdot 272''$ and (corrected) $r_4 = -2 \cdot 996''$. This lens gives a chromatic aberration of $+0 \cdot 0119$ and a spherical aberration of $-0 \cdot 028$; the respective tolerances being $\pm 0 \cdot 0228''$ and $\pm 0 \cdot 0912''$.

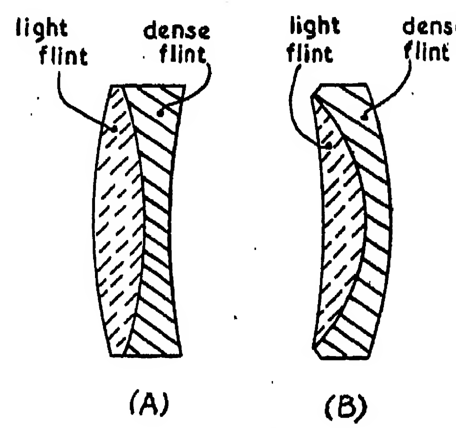


Fig. 46.

Having corrected the rear component for chromatic and spherical aberrations a precisely similar component is now used as the front lens of the system arranged symmetrically about a central diaphragm with the radii $4 \cdot 426''$ facing one another.

The separation or air space between the components has now to be decided on, and as this affects the shape of the astigmatic fields and the amount of the astigmatism it is necessary to try a number of separations. Let us choose quite arbitrarily an air space of one inch. We will now use the analytical formulæ (of page 79) to obtain the various set of aberrations given by the complete lens system. First then, an incident paraxial ray (G′-light) parallel

to the axis must be traced with $y = 0{\cdot}5$ through the six surfaces, the values from which are used in the SC′ calculation surface by surface. Calculation No. 29 illustrates the numerical work.

CALCULATION NO. 29

Spherical aberration (SC') contribution surface by surface

Surface	1*st*	2*nd*	3*rd*	4*th*	5*th*	6*th*
log ½	9·6990	9·6990	9·6990	9·6990	9·6990	9·6990
log l'	0·8857	0·9397	1·1972	1·3923*n*	1·1910*n*	0·9574
log u'	8·8133	8·7361	8·4476	8·2239*n*	8·4357*n*	8·6803
log N'	0·2145	0·1998	0·0000	0·1998	0·2145	0·0000
log i'	9·0079	9·5026	8·8554	8·8844*n*	9·4850*n*	9·2852*n*
log $(i' - u)$	9·0079	9·4033	8·2360	9·0198*n*	9·4606*n*	9·2190*n*
log $(i - i')$	8·8133	8·0251*n*	8·4222*n*	8·6510*n*	8·0223*n*	8·8760
2 colog u'_k	2·6393	2·6393	2·6393	2·6393	2·6393	2·6393
log SC'	9·0809	9·1449*n*	7·4967*n*	8·7095*n*	9·1484*n*	9·3562
SC'	+0·1205	−0·1396	−0·0031	−0·0512	−0·1407	+0·2271

Σ SC' (first component) = −0·0222. Σ SC' (second component) = +0·0352

Total sph. aberration = +0·0130.

In order to obtain numerical values for the oblique aberrations, a paraxial principal ray must then be traced from the axial centre of the diaphragm with an angle $u_{pr} = -0{\cdot}422620$ corresponding to $U_{pr} = -25°\text{–}0'\text{–}0''$. (This gives an emergent angle of approximately 21 degrees and an image height $H'_k = 4''$.) This principal ray must be traced *left-to-right* through the back component, and *right-to-left* through the front component using the same initial angle u_{pr}, thus providing the various values of i'_{pr} at each surface for use in the CC' and AC' calculations. These latter are shown in Calculations No. 30 and No. 31.

CALCULATION NO. 30

Coma Contributions (CC')

Surface	1*st*	2*nd*	3*rd*	4*th*	5*th*	6*th*
log SC'	9·0809	9·1449*n*	7·4967*n*	8·7095*n*	9·1484*n*	9·3562
log u'_k	8·6803	8·6803	8·6803	8·6803	8·6803	8·6803
log i'_{pr}	9·0769	8·2013*n*	9·5739	9·3741	8·1866*n*	9·2915
log numerator	6·8381	6·0265	5·7509*n*	6·7639*n*	6·0153	7·3280
− log i'	9·0079	9·5026	8·8554	8·8844*n*	9·4850*n*	9·2852*n*
log CC'	7·8302	6·5239	6·8955*n*	7·8795	6·5303*n*	8·0428*n*
CC'	+0·0068	+0·0003	−0·0008	+0·0076	−0·0003	−0·0110

Σ CC' (first component) = +0·0063. Σ CC' (second component) = −0·0037.

Total Coma = +0·0026.

CALCULATION No. 31

Astigmatism Contributions (AC')

Surface	1st	2nd	3rd	4th	5th	6th
$\log i'_{pr}$	9·0769	8·2013n	9·5739	9·3741	8·1866n	9·2915
$-\log i'$	9·0079	9·5026	8·8554	8·8844n	9·4850n	9·2852n
$\log\left(\frac{i'_{pr}}{i'}\right)$	0·0690	8·6987n	0·7185	0·4897n	8·7016	0·0063n
$2\log\left(\frac{i'_{pr}}{i'}\right)$	0·1380	7·3974	1·4370	0·9794	7·4032	0·0126
$+\log SC'$	9·0809	9·1449n	7·4967n	8·7095n	9·1484n	9·3562
$\log AC'$	9·2189	6·5423n	8·9337n	9·6889n	6·5516n	9·3688
AC'	+ 0·1655	− 0·0003	− 0·0858	− 0·4885	− 0·0004	+0·2338

AC' (first component) = + 0·0794. AC' (second component) = − 0·2551.

Total $AC' = -0\cdot1757$.

The Petzval curvature value (see Calculation No. 32) will be found to be $+0\cdot5166$, giving $r_{ptz} = 16\cdot77''$.

CALCULATION No. 32

Petzval Curvature (PC')

Surface	1st	2nd	3rd	4th	5th	6th
$\log N$	0·0000	0·2145	0·1998	0·0000	0·1998	0·2145
$\log N'$	0·2145	0·1998	0·0000	0·1998	0·2145	0·0000
$\log r$	0·4765	0·1045	0·6460	0·6460n	0·1045n	0·4765n
log denominator	0·6910	0·5188	0·8458	0·8458n	0·5188n	0·6910n
$\log \frac{1}{2}$	9·6990	9·6990	9·6990	9·6990	9·6990	9·6990
$+ 2\log H'$	1·2321	1·2321	1·2321	1·2321	1·2321	1·2321
$+\log N_k$	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000
$+\log(N'-N)$	9·8054	8·7373n	9·7666n	9·7666	8·7373	9·8054n
log numerator	0·7365	9·6684n	0·6977n	0·6977	9·6684	0·7365n
− log denominator	0·6910	0·5188	0·8458	0·8458n	0·5188n	0·6910n
$\log PC'$	0·0455	9·1496n	9·8519n	9·8519n	9·1496n	0·0455
PC'	1·1104	− 0·1411	− 0·7110	− 0·7110	− 0·1411	1·1104

$\Sigma\, PC'$ (first component) = + 0·2583. $\Sigma\, PC'$ (second component) = + 0·2583.

Total $PC' = +0\cdot5166$.

Determination of H'_k as used in above calculation

$l'_k = 9 \cdot 0655$	$\log - 11 \cdot 4385 = 1 \cdot 05837n$
$-l'_{pr_k} = -(-1 \cdot 373)$	$+ \log u'_{pr_k} = 9 \cdot 55770n$
$(l'_k - l'_{pr_k}) = +11 \cdot 4385$	$\log H'_k = 0 \cdot 61607$

$$H'_k = 4 \cdot 131$$

We now have sufficient information to enable the astigmatic fields and curvature of field to be drawn out to scale (see Fig. 47) as we have done by the method used in the previous examples on photographic lenses. From this it will be seen that the tangential field is almost flat and the true curvature of field curved slightly towards the lens.

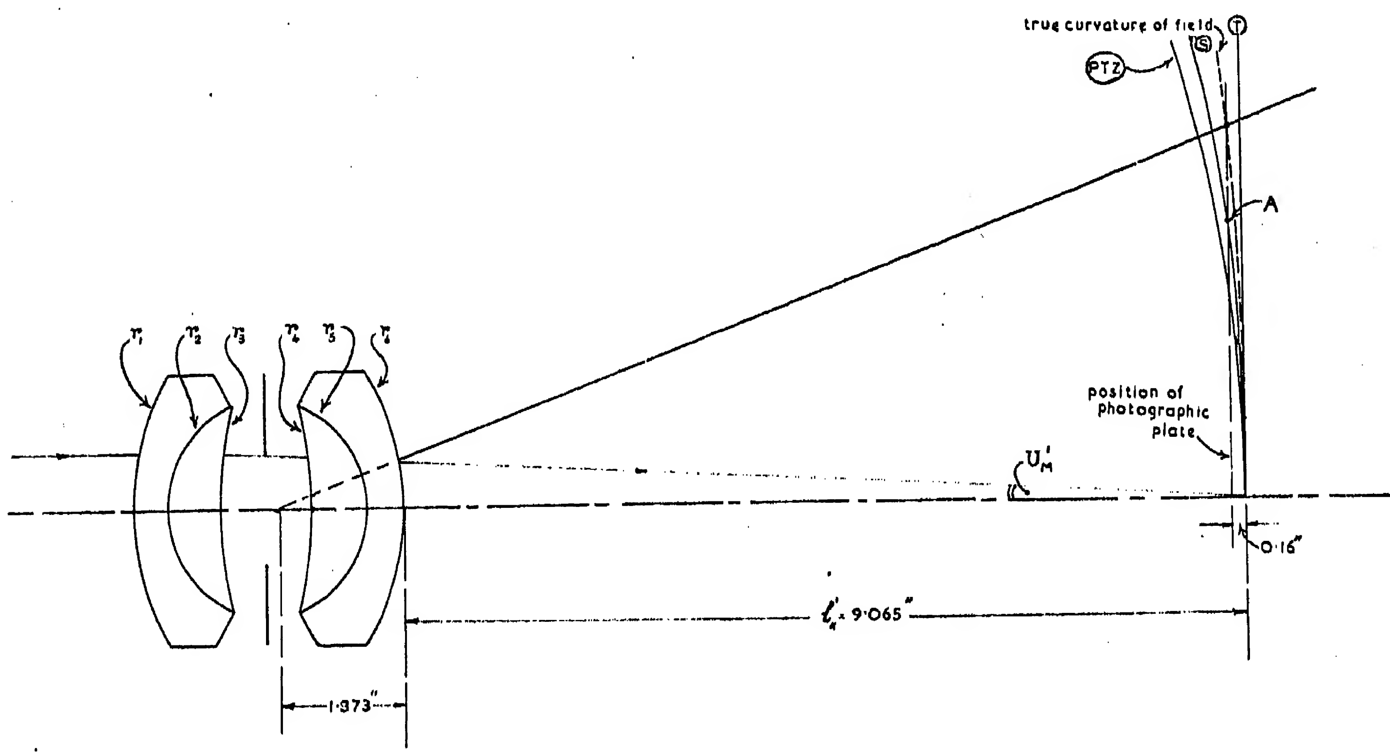


Fig. 47.

It is advisable, therefore, to try another separation of the components of the lens system in order to bend the fields backwards a little ; let us try a separation of 1·50 inches (i.e., a stop distance of 0·75 inches) and repeat the foregoing calculations. The various aberrations using this latter separation of the lenses will be found to be those shown in line 2 of the table below, where also the values for a lens separation of one inch are tabulated. The shape of the astigmatic fields for the 1·50 inch separation are shown drawn to scale in Fig. 48 from which it will be seen that the true curvature of field now bends slightly away from the lens.

Separation of lenses	SC'	CC'	AC'	Astigmatism (actual)	PC'	r_{Ptz}
1·0″	+0·0130	+0·0026	−0·1757	−0·3514	+0·5166	16·77
1·5″	+0·0029	+0·0035	−0·3146	−0·6292	+0·5166	16·77

Up till now, we have always aimed at making the discs of least confusion (situated at the mid-point between the sagittal and tangential fields) lie on a flat surface. This is quite a general practice, but it should be pointed out that some designers prefer to arrange the astigmatic surfaces in such a way that the tangential field is flat, and then to place the plate in the plane of the position of the D.L.C. at the edge of the field. Such an arrangement tends to produce a more uniform quality of definition over the whole area

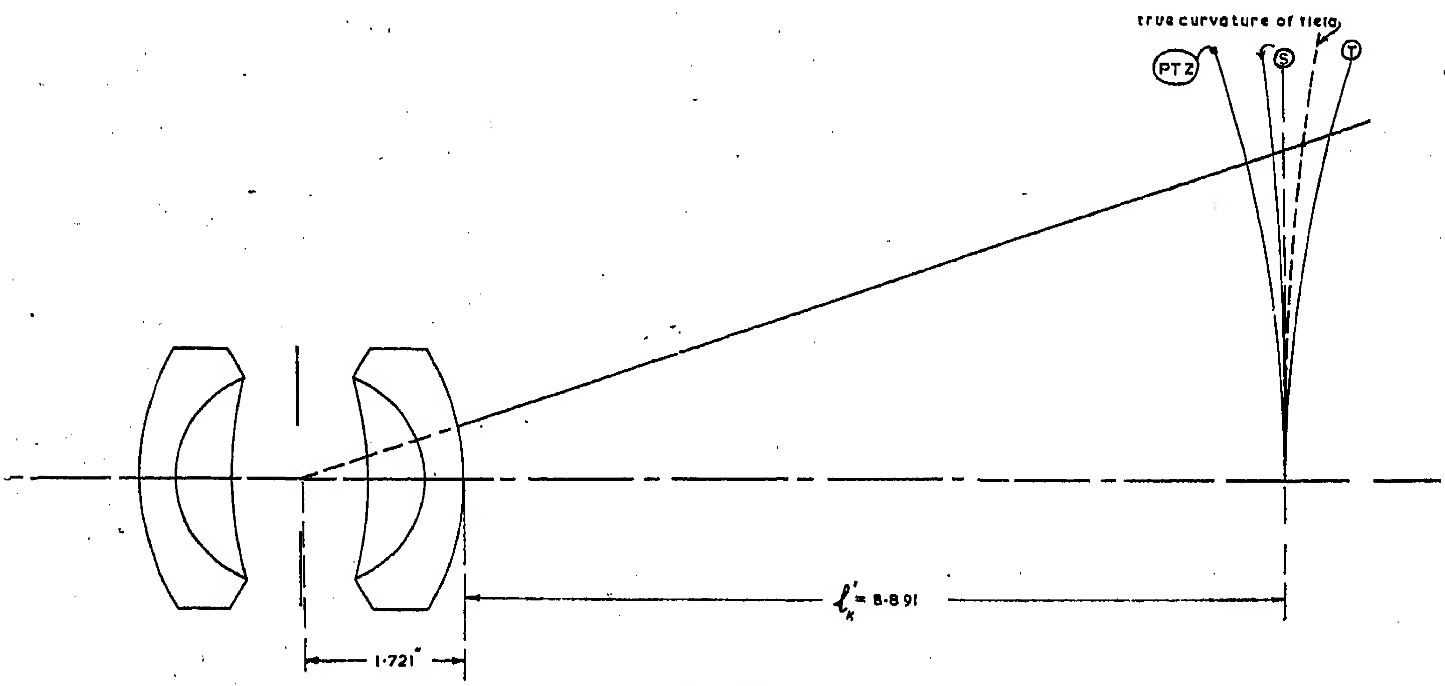


Fig. 48.

of the plate ; but on the other hand, there is a sacrifice of definition at the centre of the plate in doing this and moreover a small line image* is likely to occur at the position where the plate crosses the sagittal field surface. It therefore depends much on the requirements for which the lens is to be used, as to which criterion is to be adopted.

For the sake of our example, however, let us take in this case the solution which gives us a flat tangential field, and see what sizes the discs of least confusion are, both at the edge and centre of the field and also what length of line image is present where the flat plate crosses the sagittal surfaces.

This involves a trigonometrical ray-tracing test for the axial and oblique rays through the complete lens system in order to secure the exact values of

* If a point object were used.

the various aberrations; but we can, however, form an idea as to the quality of definition given by this particular solution of the problem, by determining the size of the discs of least confusion from the aberration values obtained from the analytical formulæ. (Although the latter give approximate values, they provide fairly close approximations to the exact trigonometrically-found aberrations for low-aperture systems.)

Taking, first, then the diameter of the D.L.C. at the edge of the field :—
(For the oblique rays) Diameter of D.L.C. = Astigmatism × $\tan U'_M$
$= 0\cdot3514 \times 0\cdot05 = 0\cdot0176''$.

N.B.—$\tan U'_M = \dfrac{\text{stop radius}}{\text{E.F.L.}}$ (approx.) $= \dfrac{0\cdot5}{10} = 0\cdot05$

(For axial rays) Referring to Fig. 47, the position of the plate is $0\cdot16''$ from the paraxial focus, and as the spherical aberration (see Calculation No. 29), is only one hundredth of an inch and therefore almost negligible, we may write

$$\text{Diameter of D.L.C.} = 2(0\cdot16 \times 0\cdot05) = 0\cdot016''.$$

The length of the line image where the plate crosses the sagittal field surface, namely at the point A (Fig. 47) will obviously be dependent on the angular cone of the emergent beam also, and therefore on the angle U'_M, so that the length of the line image $= 0\cdot16'' \times 0\cdot05 = 0\cdot016''$.

It can be seen from these various image patch sizes that the definition would be reasonably uniform over most of the plate.

As a matter of interest, we might compare these diameters with those given in the case when the true curvature of field lies on a flat plate; then, on the axis, the diameter of the D.L.C. $= 2(\frac{3}{4}$ Sph. Ab. $\times \tan U'_M) = 2(\frac{3}{4} \times 0\cdot013 \times 0\cdot05) = 0\cdot0005''$; and at the edge of the field (where the astigmatism would now be $0\cdot480''$) the diameter of the D.L.C. = Astig. $\times \tan U'_M = 0\cdot480 \times 0\cdot05 = 0\cdot024$.

Thus we have in this case extremely sharp definition at the centre and rather poor definition at the edge of the plate.

A final trigonometrical test should then be made on the complete lens system, the specification of which is given below and shown diagramatically in Fig. 47.

Front Component		*Rear Component*	
Radii	Axial thicknesses	Radii	Axial thicknesses
$r_1 = +2\cdot996''$		$r_4 = -4\cdot426''$	
	$d'_1 = 0\cdot40''$		$d'_4 = 0\cdot60''$
$r_2 = +1\cdot272''$		$r_5 = -1\cdot272''$	
	$d'_2 = 0\cdot60''$		$d'_5 = 0\cdot40''$
$r_3 = +4\cdot426''$		$r_6 = -2\cdot996''$	

Air separation of the lenses$=1\cdot00''$, with diaphragm placed at mid point between them.

The glasses are those already given on page 111.

Symmetrical Type ("new" glass)

It was shown in the section dealing with achromatized meniscus lenses, that astigmatism could be reduced by employing two glasses which had considerable difference in N value and small difference in V value, and moreover that the tangential field could be moved either in front of or away from the sagittal field. Thus it was quite naturally assumed that by using two "new glass" achromats symmetrically about a central diaphragm a better state of correction for astigmatism might be hoped for than in the case of the "old glass" symmetrical type. It is advisable, therefore, to satisfy oneself about this by carrying out some numerical examples, which at the same time provide accumulated experience and information which stand us in good stead for designing purposes later.

We will ask that the lens system has an equivalent focal length of ten inches, a full aperture of one inch, and that it shall cover a half-plate (i.e., an 8 inch diagonal or $H_k'=4''$).

Recalling the earlier remarks connected with the design of symmetrical types, we will use the following glasses :—

	N_D	$(N_{G'}-N_D)$	$N_{G'}$	V	V difference
Extra Light Flint	1·53160	0·01400	1·54560	49·0	7·9
Dense Barium Crown	1·61400	0·01382	1·62782	56·9	

First, the curvature of each component for achromatism should be obtained from :—

$$R_a=\frac{1}{f'\,(V_a-V_b)\delta N_a} \quad \text{and} \quad R_b=\frac{1}{f'\,(V_b-V_a)\delta N_b}$$

N.B.—From thin lens formulæ, we may assume the equivalent focal length f' of each component as 20 inches; but as before it is found necessary to make this slightly less (say 15″) in order to secure the correct curvatures.

Thus, for the first lens of the system (see Fig. 49) $R_a=+0\cdot6106$ and $R_b=-0\cdot6028$, and for the second lens $R_a=-0\cdot6028$ and $R_b=+0\cdot6106$.

The analytical G-sum formulæ are then used for the determination of the spherical aberration for the front and rear component in turn ; the result of such calculations give the following equations :—

Front Component

$$\text{Sph. Ab. (lens } a) = y^2\left\{17\cdot81 + 9\cdot45R_2 + 170\cdot89R_2^2\right\}$$

$$\text{Sph. Ab. (lens } b) = y^2\left\{-12\cdot91 + 3\cdot93R_3 - 150\cdot90R_3^2\right\}$$

For cemented contact (i.e., with $R_3 = R_2$)

$$\text{Total Sph. Ab.} = y^2\left\{4\cdot90 + 13\cdot38R_2 + 19\cdot99R_2^2\right\}$$

Rear Component

$$\text{Sph. Ab. (lens } a) = y^2\left\{-14\cdot14 + 19\cdot66R_2 - 150\cdot90R_2^2\right\}$$

$$\text{Sph. Ab. (lens } b) = y^2\left\{20\cdot24 - 36\cdot40R_3 + 170\cdot89R_3^2\right\}$$

$$\text{Total Sph. Ab.} = y^2\left\{6\cdot10 - 16\cdot74R_2 + 19\cdot99R_2^2\right\}$$

Putting $y = 0\cdot5$ (the semi-aperture) and substituting various values for R_2; we get the results tabulated below :—

Front Component

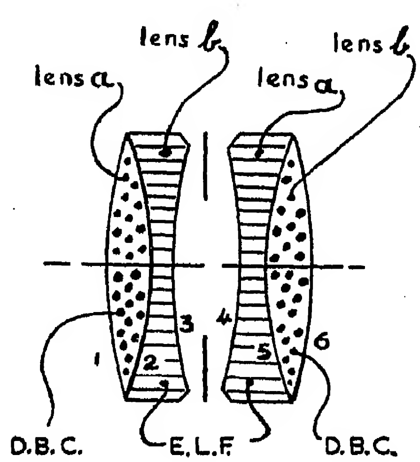
For R_2	=	$-0\cdot10$	$-0\cdot20$	$-0\cdot30$	$-0\cdot40$	$-0\cdot50$	$-0\cdot60$
Sph. Ab.	=	$+0\cdot940$	$+0\cdot756$	$+0\cdot671$	$+0\cdot686$	$+0\cdot802$	$+1\cdot017$

Rear Component

For R_2	=	$+0\cdot10$	$+0\cdot20$	$+0\cdot30$	$+0\cdot40$	$+0\cdot50$	$+0\cdot60$	$+0\cdot70$
Sph. Ab.	=	$+1\cdot156$	$+0\cdot888$	$+0\cdot719$	$+0\cdot650$	$+0\cdot682$	$+0\cdot813$	$+1\cdot044$

These results are plotted in Fig. 50.

Fig. 49.



$r_1 = +3\cdot838''$ $r_4 = -3\cdot956''$
$d'_1 = 0\cdot30''$ $d'_4 = 0\cdot16''$
$r_2 = -2\cdot875''$ $r_5 = +2\cdot857''$
$d'_2 = 0\cdot16''$ $d'_5 = 0\cdot30''$
$r_3 = +3\cdot956''$ $r_6 = -3\cdot838''$

It will be noted that the spherical aberration curves never reach a zero value; in fact this aberration has a very considerable positive value for either of the components. The parabolas, however, are at much the same height from the abscissa, and therefore one may choose a similar contact curvature (namely $R_2 = 0\cdot35$) for nearly minimum spherical aberration and which will give two exactly similar lenses for the complete symmetrical system.

The latter is illustrated in Fig. 49 where the radii and axial thicknesses are shown. In this example, we will now find the separation of the lenses to employ by determination of the correct diaphragm position for the elimination of coma. This may be carried out in the first instance by utilizing the l'_{EP} equation (given on page 109) with the accompanying ray-trace to give the real position of the diaphragm, followed by a test with the usual CC' calculation on the complete lens system arranged symmetrically about the stop. Should the coma exceed the tolerance, adjustment of the stop-position must be made.

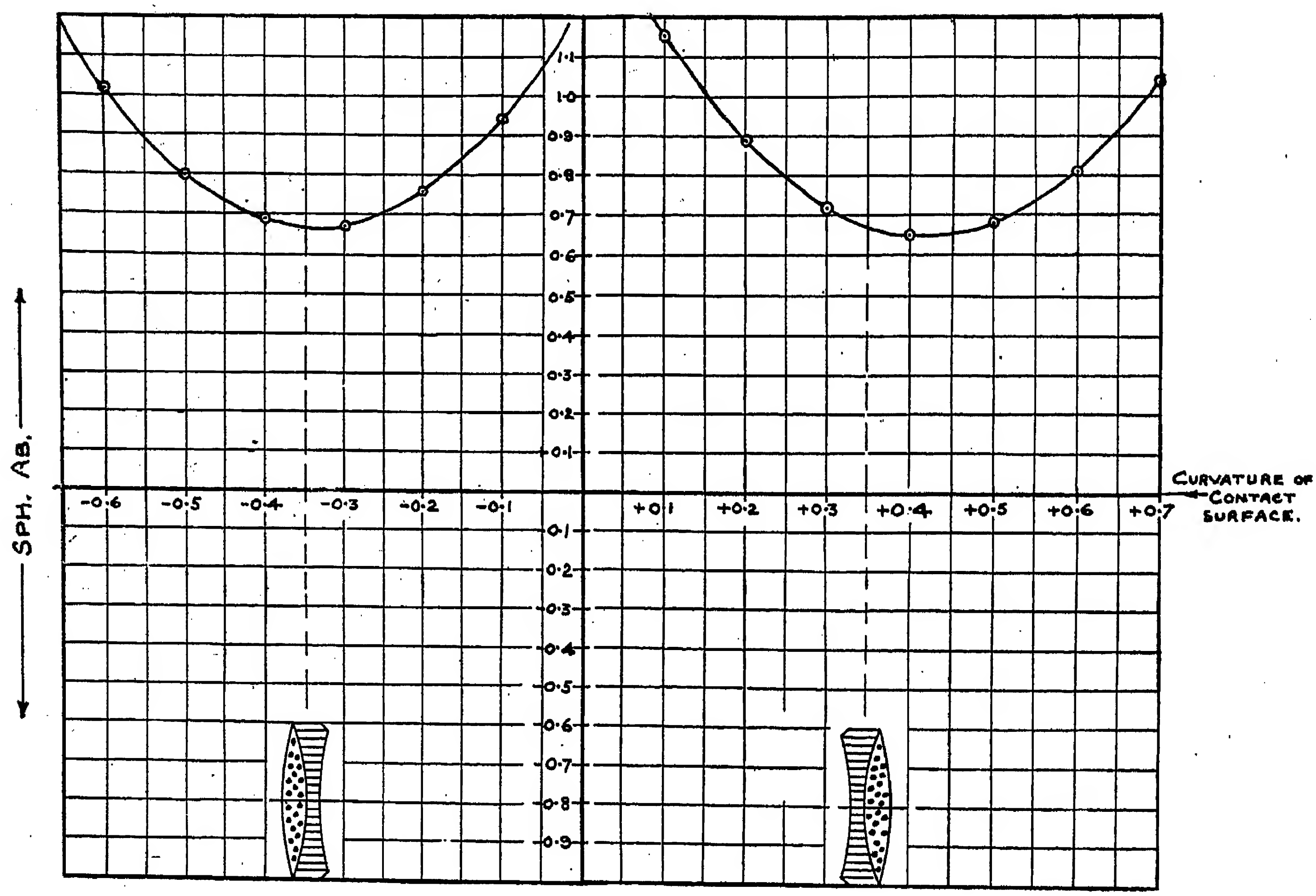


Fig. 50.

Using this specification it is now of interest to find the shape of the astigmatic surfaces; we can do this with sufficient accuracy at first by employing the formulæ for the various aberration contributions SC', CC', PC', AC'. The following results will be obtained :—

$SC' = +0 \cdot 2106$; $CC' = -0 \cdot 0014$; $PC' = 0 \cdot 2770$; $r_{ptz} = 22 \cdot 17$; $AC' = 0 \cdot 4291$.
Astigmatism (actual) $= 0 \cdot 8582$ at $23\frac{1}{3}$ degrees.

With the H'_k obtained in the PC′ calculation and the l'_{prk} obtained in the oblique principal ray trace, the inclination of the emergent ray from the lens system may be drawn out to scale ; the Petzval surface can then be put

in, and the AC' length measured off, together with twice the AC' value. These last two points will give the positions of the sagittal and tangential fields respectively. Such a drawing is shown to scale in Fig. 51 where it will be noticed that the astigmatic surfaces are heavily curved towards the lens, and

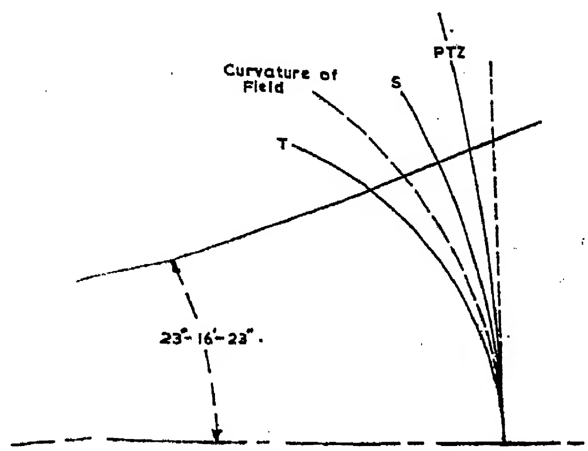
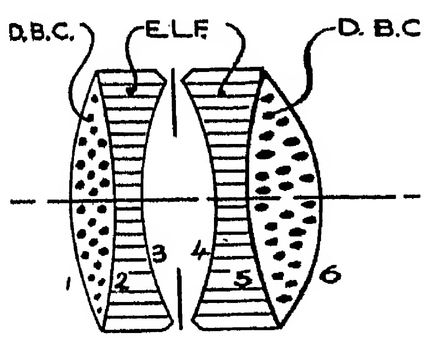


Fig. 51.

there is a large amount of astigmatism present. We must attempt to improve this, and will therefore choose another shape of the lenses (indicated by the graphs of Fig. 50) with a contact curvature of say 0·20. This specification is shown in Fig. 52. If this is tested out as before, we find the $SC' = +0\cdot2975$; $CC' = -0\cdot0004$; $PC' = 0\cdot1960$; $r_{ptz} = 22\cdot09$; $AC' = 0\cdot1442$; Astigmatism (actual) $= 0\cdot2884$ at $22\frac{1}{2}$ degrees.

Fig. 52.



$r_1 = +2\cdot435''$
$d'_1 = 0\cdot30''$
$r_2 = -5\cdot000''$
$d'_2 = 0\cdot20''$
$r_3 = +2\cdot483''$

$r_4 = -2\cdot483''$
$d'_4 = 0\cdot20''$
$r_5 = +5\cdot000''$
$d'_5 = 0\cdot30''$
$r_6 = -2\cdot435''$

It will be noted that the astigmatism is less (about 1/3 less) than in the previous case (although the spherical aberration has increased) but the S and T fields are still curved towards the lens system (see Fig. 53).

A further shape of the lenses must therefore be investigated and this time we will choose $R_2 = 0\cdot05$, thus giving us the three "bendings" with R_2 equal in turn to 0·35, 0·20 and 0·05. This last named shape of the lenses is depicted in Fig. 54 where the specification is also given.

When this is tested out, the following results will be found:—

$SC' = +0\cdot4280$; $CC' = +0\cdot0026$; $r_{ptz} = 22\cdot14''$; $AC' = -0\cdot0501$; Astigmatism (actual) $= -0\cdot1002$ at 22 degrees semi-field. It will be noted from

these that the AC' value is now not only small but that it has a negative sign. This indicates that the sagittal focal surface is to the right of the Petzval surface as will be seen when the astigmatic surfaces are drawn out to scale (see Fig. 55). The tangential surface is also to the right of the Petzval surface, and the fields have thus swung over to positions opposite to those indicated in Fig. 51 and Fig. 53; also the tangential field is practically flat.

The astigmatism (actual) is only 0·10 inch which is approximately $3\frac{1}{2}$ times smaller than the value for this aberration given by the "old glass" symmetrical type (in the previous section) of similar aperture and angular field; unfortunately this good point in favour of the "new glass" symmetrical lens is somewhat off-set by the relatively large amount of spherical aberration which has to be introduced in order to accomplish this.

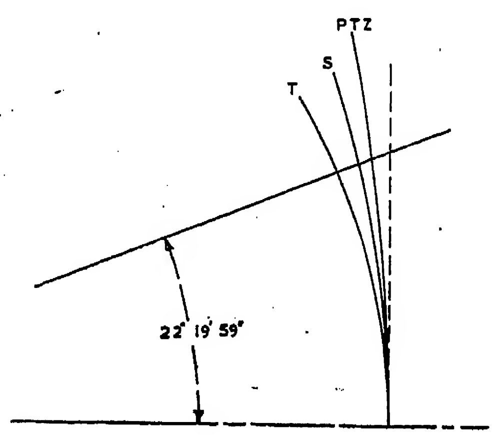


Fig. 53.

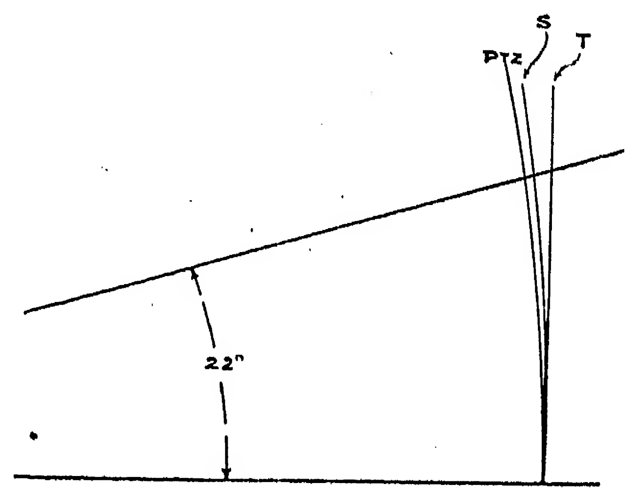
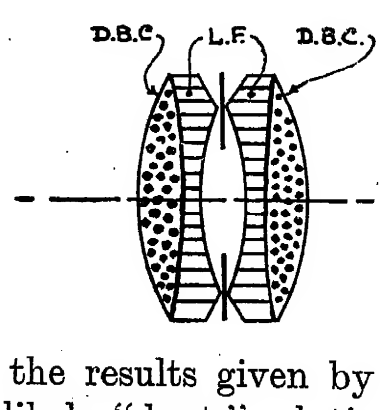


Fig. 55.

Although the definition of the image would suffer on account of this considerable amount of spherical aberration, it is of interest to note the reduction in the astigmatism with this type of lens system.

Fig. 54.



$r_1 = 1 \cdot 784''$
$d'_1 = 0 \cdot 30''$
$r_2 = -20 \cdot 000''$
$d'_2 = 0 \cdot 16''$
$r_3 = +1 \cdot 809''$

$r_4 = -1 \cdot 809''$
$d'_4 = 0 \cdot 16''$
$r_5 = +20 \cdot 000''$
$d'_5 = 0 \cdot 30''$
$r_6 = -1 \cdot 784''$

From the results given by the three foregoing trials it will be possible to select a likely "best" solution (which obviously will not be widely different from that given by shape No. 3) and this solution would have to be tested trigonometrically for chromatic aberration, spherical aberration, coma, astigmatism at two or more inclinations to the axis, and then the discs of least confusion calculated.

Anastigmat Lenses (Rudolph type)

It will be noticed from the foregoing examples of the symmetrical type of photographic lens that the "old glass" form gives a result with a considerable amount of astigmatism present but with a small amount of spherical aberration; whilst the form employing two "new glass" components gives small amounts of astigmatism and large amounts of spherical aberration. Also it will be recalled from the examples already given for the "old glass" and "new glass" meniscus landscape lenses that somewhat similar circumstances also occur. It was quite natural therefore that an attempt should be made to combine one component of each type with a view to decreasing the astigmatism and spherical aberration simultaneously with a resultant better all-round performance of the lens system.

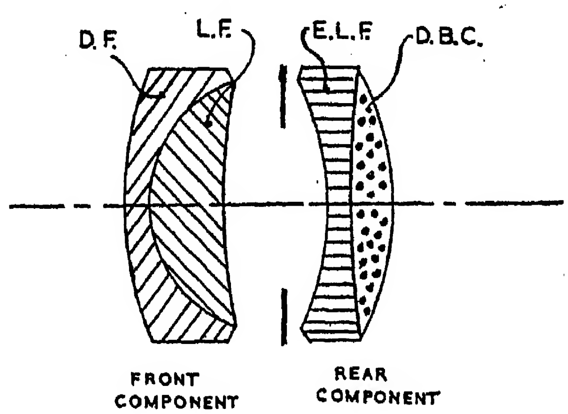


Fig. 56.

Such a lens was designed by Rudolph about 1890 and it is undoubtedly an interesting type; moreover, it follows as a natural sequence in the order of designing photographic lenses chosen in this chapter. We will therefore attempt to formulate a systematic method of designing what may now be called the Rudolph type of anastigmat.

The general outline of the designing process in this case, is to employ an "old glass" achromatic component and a "new glass" achromat arranged as in Fig. 56 which is diagrammatic only. As the "new glass" achromat nearly always produces an under-corrected (i.e., positive) amount of spherical aberration it is necessary to employ suitable glasses for the front component which will produce over-corrected (or negative) spherical aberration, in order to give freedom from this aberration when the two components are placed together. Furthermore, it will be recalled that in order to keep the astigmatism of each component down to within controllable limits it is possible to tolerate a large amount of coma in each if the amounts balance out when the two components are placed together. Both these requirements can be met by employing pairs of glasses which have a small difference in V value and as large an N difference as possible.

Let us therefore commence the design by choosing such pairs of glasses; namely,

Front component :—	N_D	$N_{G'}$	V	$\delta N = (N_{G'} - N_D)$
Dense Flint ..	1·61820	1·64060	36·4	0·02240
Light Flint ..	1·57460	1·59280	41·4	0·01820

Rear component :—	N_D	$N_{G'}$	V	$\delta N = (N_{G'} - N_D)$
Extra Light Flint	1·53160	1·54560	49·0	0·01400
Dense Barium Crown ..	1·61400	1·62782	56·9	0·01382

We will ask that the specification for the complete lens system shall be :—
Equivalent focal length = 10 inches ; Full aperture = 1 inch. To cover a half-plate ($6\frac{1}{2}'' \times 4\frac{3}{4}''$), i.e., $H'_k = 4''$.

The design will be commenced by calculating the total curvature $\mathcal{R}$ of each lens of each component, from :—

$$\mathcal{R}_a = \frac{1}{f'(V_a - V_b)\,\delta N_a} \quad \text{and} \quad \mathcal{R}_b = \frac{1}{f'(V_b - V_a)\,\delta N_b}$$

Putting in the appropriate reduced values of f' in order to secure an equivalent focal length of twenty inches for each component, we find

for the Front Component $\left\{ \begin{array}{l} \mathcal{R}_a = -0{\cdot}6398 \\ \mathcal{R}_b = +0{\cdot}7885 \end{array} \right.$ and for the Rear Component $\left\{ \begin{array}{l} \mathcal{R}_a = -0{\cdot}6028 \\ \mathcal{R}_b = +0{\cdot}6106 \end{array} \right.$

We must now determine the spherical aberration for various shapes of each component for incident parallel light in both cases. This is done by employing the analytical G-sum equations (page 112) using the appropriate values for N_a and N_b ; $\mathcal{R}_a$ and $\mathcal{R}_b$; $v'_2 = v_3$; and $(l'_b)^2$.

On working out these G-sum calculations, we find for the *front* component :

$$\text{Total spherical aberration} = y^2\left\{18{\cdot}61 - 50{\cdot}94R_2 + 28{\cdot}98R_2^2\right\}$$

and for the *rear* component :—

$$\text{Total spherical aberration} = y^2\left\{6{\cdot}10 - 16{\cdot}74R_5 + 19{\cdot}99R_5^2\right\}$$

Putting the semi-aperture value (y) equal to 0·50 inch in the above equations and substituting suitable values for the contact curvatures R_2 and R_5, we get the following tabulated results :—

Front Component

Curvature R_2 =	+0·5	+0·6	+0·7	+0·8	+0·9	+1·0	+1·1	+1·2
Sph. Ab. =	+0·096	−0·380	−0·712	−0·899	−0·940	−0·837	−0·589	−0·197

Rear Component

Curvature R_5 =	+0·1	+0·2	+0·3	+0·4	+0·5	+0·6	+0·7
Sph. Ab. =	+1·156	+0·888	+0·719	+0·650	+0·682	+0·813	+1·044

These figures are now plotted as shown in Fig. 57 and if we considered these curves *alone* the corresponding contact curvatures (plotted as abscissa) could be chosen where the spherical aberration of the front component balances out with that of the rear component; for example, where $R_2 = +1 \cdot 02$ in conjunction with $R_5 = +0 \cdot 25$, or where $R_2 = +0 \cdot 75$ in conjunction with $R_5 = +0 \cdot 61$.

But the more important part of the procedure has now to be undertaken, namely, that of determining the astigmatism of each component for various bendings. This entails the tracing of a principal ray obliquely through both the components for a given separation of the lenses. The separation* will be obtained by determining the diaphragm position (giving reduction of coma) for each component individually, and adding these. An oblique principal ray must then be traced right-to-left and left-to-right from the centre of the diaphragm at (say) an inclination of − 22 degrees through each lens and the astigmatism calculated. If this is done for the case when $R_2 = +0 \cdot 75$ and $R_5 = +0 \cdot 61$ it will be found that the astigmatism given is too excessive to be handled conveniently, and therefore we will try the other two shapes (given above) namely with $R_2 = +1 \cdot 02$ and $R_5 = +0 \cdot 25$. These last-named lens-shapes do give less astigmatism although the back component still introduces a large value for this aberration. This indicates that we ought to "bend" the lens components with a view to ascertaining the way in which the astigmatism may be varied.

The choice of suitable "bendings" to take might be difficult without some previous experience; but as the astigmatism (for both these components) changes very rapidly both in quantity and sign for a slight change in the curvature of the contact surface; it will be found advisable to use the following suggested values for R_2 and R_5:—

Front component: $R_2 = +0 \cdot 95$; $+1 \cdot 00$; and $+1 \cdot 05$.
Rear component: $R_5 = +0 \cdot 20$; $+0 \cdot 15$; and $+0 \cdot 10$.

The respective radii of these lenses utilizing these contact curvatures and the total curvatures R_a and R_b already determined, will therefore be:—

Front Component

$R_2 = +0 \cdot 95$	$R_2 = +1 \cdot 00$	$R_2 = +1 \cdot 05$
$r_1 = +3 \cdot 224''$	$r_1 = +2 \cdot 776''$	$r_1 = +2 \cdot 438''$
$r_2 = +1 \cdot 053''$	$r_2 = +1 \cdot 000$	$r_2 = +0 \cdot 952$
$r_3 = +5 \cdot 000''$	$r_3 = +3 \cdot 733''$	$r_3 = +3 \cdot 146''$

Axial thicknesses, $d_1' = 0 \cdot 21$ and $d_2' = 0 \cdot 54$ (from a scale drawing, Fig. 56).

* For simultaneous reduction of coma and astigmatism, the CC' and AC' equations indicate stop distances of $+0 \cdot 40$ for the front component and $-0 \cdot 25$ for the rear component.

Rear Component

$R_5 = +0\cdot10$	$R_5 = +0\cdot15$	$R_5 = +0\cdot20$
$r_4 = -1\cdot988$	$r_4 = -2\cdot209$	$r_4 = -2\cdot483$
$r_5 = +10\cdot000$	$r_5 = +6\cdot667$	$r_5 = +5\cdot000$
$r_6 = -2\cdot150$	$r_6 = -2\cdot371$	$r_6 = -2\cdot648$

Axial thicknesses : $d_4' = 0\cdot20$, $d_5' = 0\cdot30$.

For each of the above six lens shapes, an oblique principal ray trace and the astigmatism calculation must now be carried out. This will, of course, take time but it is the only satisfactory way of carrying out the design in a systematic manner. The results of such calculations will be found to be as follows :—

Front Component			*Rear Component*		
Lens Shape	*Astigmatism*	X_t'	*Lens Shape*	*Astigmatism*	X_t'
$R_2 = +0\cdot95$	$+1\cdot020$	$+2\cdot323$	$R_5 = +0\cdot10$	$-0\cdot312$	$-0\cdot369$
$R_2 = +1\cdot00$	$+0\cdot098$	$+0\cdot901$	$R_5 = +0\cdot15$	$+0\cdot250$	$+0\cdot521$
$R_2 = +1\cdot05$	$-0\cdot540$	$-1\cdot380$	$R_5 = +0\cdot20$	$+0\cdot768$	$+1\cdot401$

It will be noted that in the foregoing table an additional column gives the values $X_t' = (l' - l_t')$; that is, the distance of the tangential field focus (at 20 degree inclination) from the paraxial focus. It is of interest to know this if one is attempting to obtain a flat tangential field, the criterion which is frequently used for giving the most uniform definition over the whole area of the plate.

All these various values are now plotted on the same graph as the spherical aberration (Fig. 57) and the curves drawn in. Thus the extent of the different aberrations may be seen simultaneously.

In order to arrive at the most suitable shape of each component when combined as one lens system, it is necessary to assess the pros and cons of the various aberrations in each case and then to decide on the best compromise.

For example, a very natural first choice might be that of taking the positions where the X_t' curves cross the abscissa which thus should give a flat tangential field when the two components are combined. Referring to the graph, these positions indicate a contact curvature of $+1\cdot02$ for the front component, and $+0\cdot12$ for the rear component. Looking now at the astigmatism corresponding to these positions, we find that the front component gives a value for this aberration of $-0\cdot20$ whilst that of the rear component is $-0\cdot10$, giving a total value of $-0\cdot30$ for the complete lens. Although the astigmatism is thus rather high, it has the correct sign ; for it will be remembered (from the previous examples) that there must be a certain amount of *negative* astigmatism in order to secure a flat field for the image surface. The corresponding amounts of spherical aberration indicate $-0\cdot80$ for the

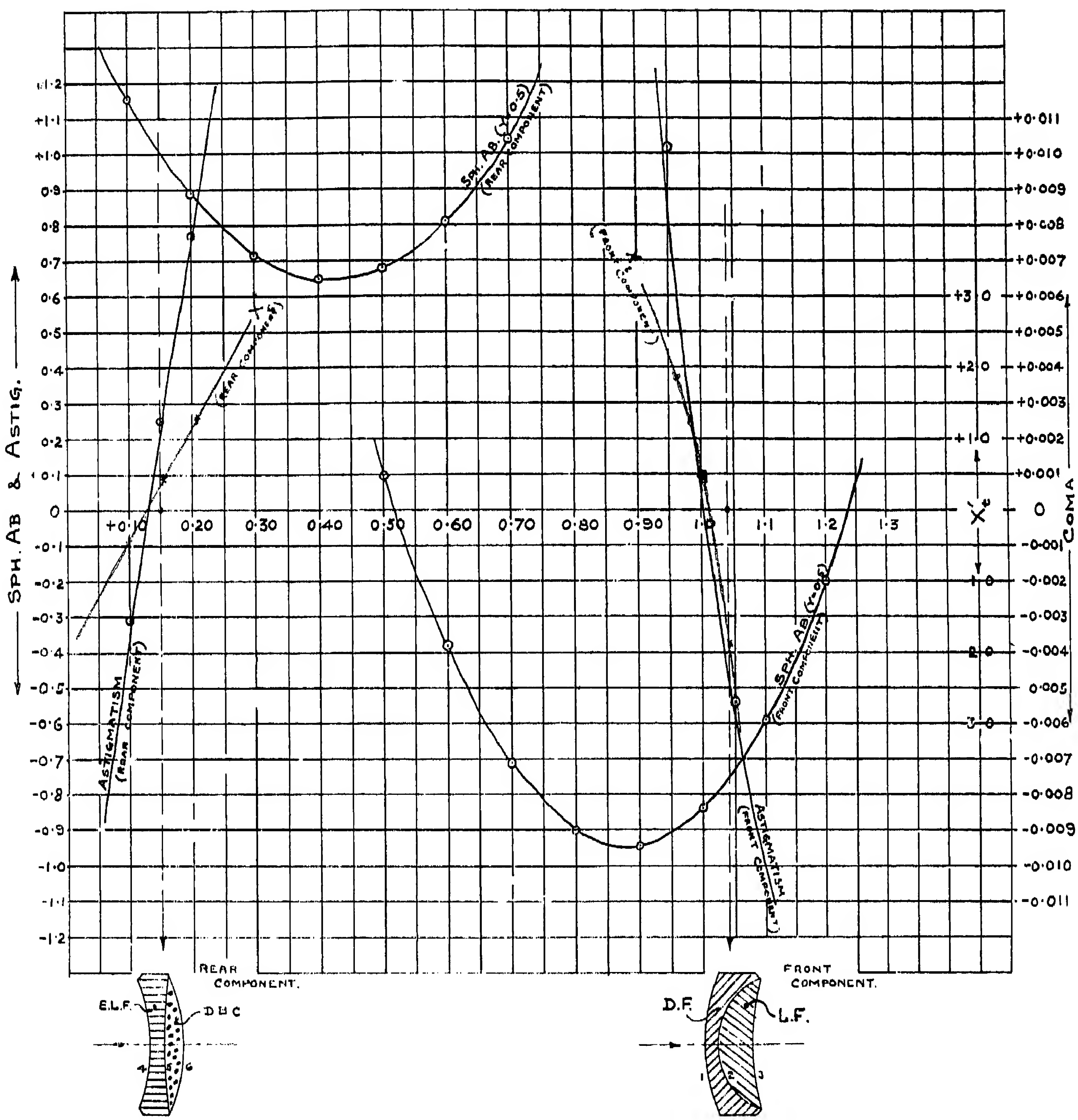

SPH. AB & ASTIG.
COMA
SPH. AB. (Y=0·5) (REAR COMPONENT)
ASTIGMATISM (REAR COMPONENT)
ASTIGMATISM (FRONT COMPONENT)
SPH. AB. (Y=0·5) (FRONT COMPONENT)
REAR COMPONENT.
FRONT COMPONENT.
E.L.F.
D.B.C
D.F.
L.F.

Fig. 57.

front component and $+1\cdot 09$ for the rear component, giving a total of $+0\cdot 29$ (i.e., rather heavily undercorrected).

Another likely solution might be that when $R_2 = +1\cdot 04$ and $R_5 = +0\cdot 17$; for here we see from the graph that the X_t' values for the front and rear components are $-0\cdot 20$ and $+0\cdot 20$ respectively and therefore balance out, whilst at the same time the astigmatism almost cancels out (i.e., $-0\cdot 43$ for the front component and $+0\cdot 48$ for the rear component).

Students could quite profitably carry out trigonometrical tests on either or both of these solutions, and the following specifications can be prepared :— *First* solution, with $R_2 = +1\cdot 02$ and $R_5 = +0\cdot 12$.

Front component		*Rear component*	
$r_1 = +2\cdot 630$		$r_4 = -2\cdot 071$	
	$d_1' = 0\cdot 21$		$d_4' = 0\cdot 20$
$r_2 = +0\cdot 980$		$r_5 = +8\cdot 333$	
	$d_2' = 0\cdot 54$		$d_5' = 0\cdot 30$
$r_3 = +4\cdot 320$		$r_6 = -2\cdot 038$	

r_3 and r_6 to be corrected for achromatism.

Second solution, with $R_2 = +1\cdot 04$ and $R_5 = +0\cdot 17$.

Front component		*Rear component*	
$r_1 = +2\cdot 499$		$r_4 = -2\cdot 311$	
	$d_1' = 0\cdot 21$		$d_4' = 0\cdot 20$
$r_2 = +0\cdot 962$		$r_5 = +5\cdot 882$	
	$d_2' = 0\cdot 54$		$d_5' = 0\cdot 30$
$r_3 = +3\cdot 976$		$r_6 = -2\cdot 270$	

r_3 and r_6 to be corrected for achromatism.

The procedure is to trace D and G′ rays at $\sqrt{0\cdot 5}$ of the semi-aperture parallel to the axis correcting the radius r_3 to give complete achromatism for the front component, then to continue the ray-tracing through the second component and to adjust r_6 so that the whole lens system is achromatized. A marginal and a paraxial ray in G′-light is then traced right through the complete lens and the spherical aberration ascertained.

Using the correct stop distances an oblique principal ray (at an inclination of -22 degrees to the axis) must be traced *right-to-left* through the front component and *left-to-right* through the rear component; the angle values from these ray-tracings are then utilized in the astigmatism calculations (by s and t formulæ). The astigmatism produced by the front component is, of course, carried on to the second lens and the final l_{s6}' and l_{t6}' then obtained.

The first important point one wishes to know at this stage is whether the l_{t6}' agrees with the l_6' (i.e., if a flat tangential field is desired), or if there is

some spherical aberration present, whether the disc of least confusion for the axial rays coincides with the D.L.C. for the oblique rays (i.e., the mid-point between l'_s and l'_t). Assuming that the field does appear reasonably flat for this inclination of the *pr* ray (namely, about 20 degrees) to the axis; another inclination of say 14 degrees must be taken and the astigmatism calculated, the shape of the astigmatic fields then being drawn out to scale.

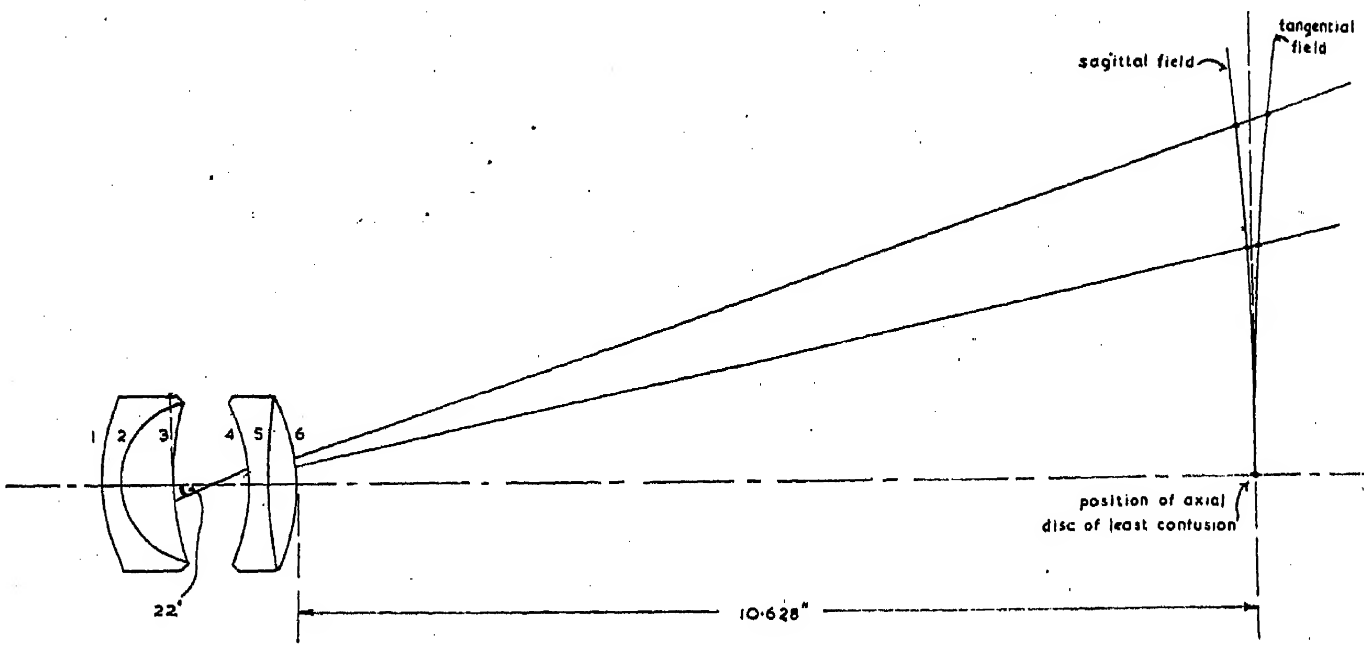


Fig. 58.

Having placed the flat plate at the mid-point between s'_{6pr} and t'_{6pr} for the extreme marginal inclination, the sizes of the discs of least confusion can be calculated at this position, at the 14 degree inclination, and on the axis. Thus an estimation of the quality of the definition may be obtained over the whole field. Finally, the exact amount of the coma may be determined by tracing *a* and *b* rays (see Fig. 32) through the complete system, and combining these calculations with the already traced *pr* ray. The method for doing this has been explained in previous sections (see page 88).

The author has, however, chosen a solution intermediate between the two mentioned above (namely with $R_2 = +1\cdot044$ and with $R_5 = +0\cdot150$) and has tested this trigonometrically throughout in order to serve as a guide for those who wish to carry out a similar test. The specification in this case is as follows:—

$r_1 = +2\cdot475$ | $r_4 = -2\cdot209$

$d'_1 = 0\cdot21$ | $d'_4 = 0\cdot20$

$r_2 = +0\cdot958$ | $r_5 = +6\cdot667$

$d'_2 = 0\cdot54$ | $d'_5 = 0\cdot30$

$r_3 = +3\cdot208$ | $r_6 = -2\cdot357$

r_3 and r_6 to be corrected for achromatism.

With this design, the *final* results of the trigonometrical test will be found to be as follows :—

Chromatic aberration $= + 0 \cdot 0018$ Chrom. Ab. Tolerance $= \pm 0 \cdot 010$

Spherical aberration $= - 0 \cdot 0423$ Sph. Ab. Tolerance $= \pm 0 \cdot 040$

$$\begin{cases} l'_6 = 10 \cdot 5668 ; \\ L'_6 = 10 \cdot 6091 ; \end{cases} \qquad \begin{aligned} u' &= 0 \cdot 043019 \\ U' &= 2^\circ\text{–}27'\text{–}48''. \end{aligned}$$

Astigmatism :— $l'_{s6} = 10 \cdot 4883$; $l'_{t6} = 10 \cdot 7678.$

(at 20° obliquity) Astig. $= - 0 \cdot 2795.$

Astigmatism :— $l'_{s6} = 10 \cdot 5272$; $l'_{t6} = 10 \cdot 6278.$

(at 13° obliquity) Astig. $= - 0 \cdot 1006.$

$\text{Coma}'_T = + 0 \cdot 026 \cdot$ $\text{Coma}'_S = + 0 \cdot 008.$

(For shape of astigmatic fields see Fig. 58.)

Position and Size of Discs of Least Confusion :—

All discs of least confusion situated at $10 \cdot 628''$ from pole of last surface (i.e., mid-point between l'_{s6} and l'_{t6} at extreme obliquity).

Diameter of D.L.C. (at 20° obliquity) = Astig. $\times \tan U'_M = 0 \cdot 0120''$

Diameter of D.L.C. (at 13° obliquity) = Astig. $\times \tan U'_M = 0 \cdot 0043''$

Diameter of D.L.C. on the axis $= 0 \cdot 019 \times \tan U'_M = 0 \cdot 0008''$

Petzval curvature $R_{ptz} = 0 \cdot 0313$ (i.e., $r_{ptz} = 31 \cdot 95$).

Distortion $= - 0 \cdot 0094.$

On looking at the above results and comparing them with those given by the "old" glass and "new" glass symmetrical types, the main advantage of being able to keep down to within small limits both the spherical aberration and astigmatism simultaneously will be clearly brought out by this type of lens. In this particular example, however, the final coma value is rather high and a little too large to be tolerated. An attempt therefore to reduce this aberration should be made, by a movement of the diaphragm.

It will be realized, of course, that only the general outline of the procedure can be given in a book of this kind. Obviously, such a design is a complicated one and requires time and patience to be spent on the refinement of the process given above in order to secure the best possible result.

Anastigmat (Cooke type)

The Cooke lens (originally designed by H. Dennis Taylor) is so widely known and has such a high reputation for performance, that it was felt desirable to include it as one of the examples of an anastigmatic lens in this chapter on photographic lenses.

The general principles involved in the design of this lens were based on novel means for securing little astigmatism and a flat field, with a consequent improvement in the quality of definition over a larger angular field and for a larger aperture of the lens.

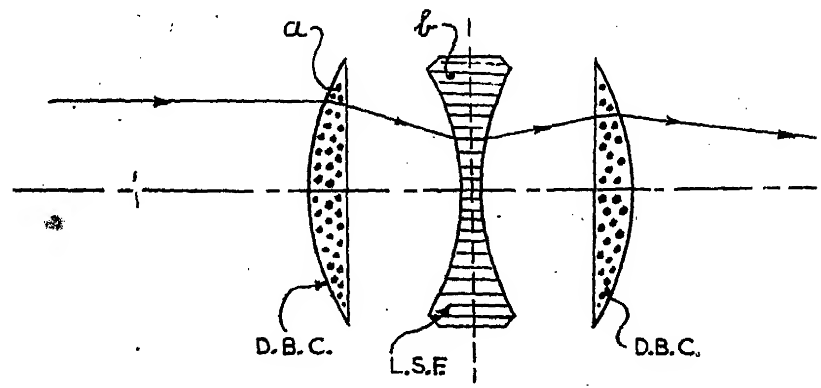


Fig. 59.

The method of reducing the Petzval sum (i.e., increasing the Petzval curvature) suggested by Taylor was to introduce some diverging lens system *between* two converging lens systems and at the same time to have considerable air-spacing between the components. The presence of such an intermediate concave lens not only gives a Petzval value opposite in sign to that given by the convex lenses, but owing to the fact that there is a considerable air-space between the lenses the rays are made to *drop towards the concave surfaces and rise towards the convex surfaces* (see Fig. 59); this enables steeper concave surfaces to be utilized. The resultant effect is that the negative contribution

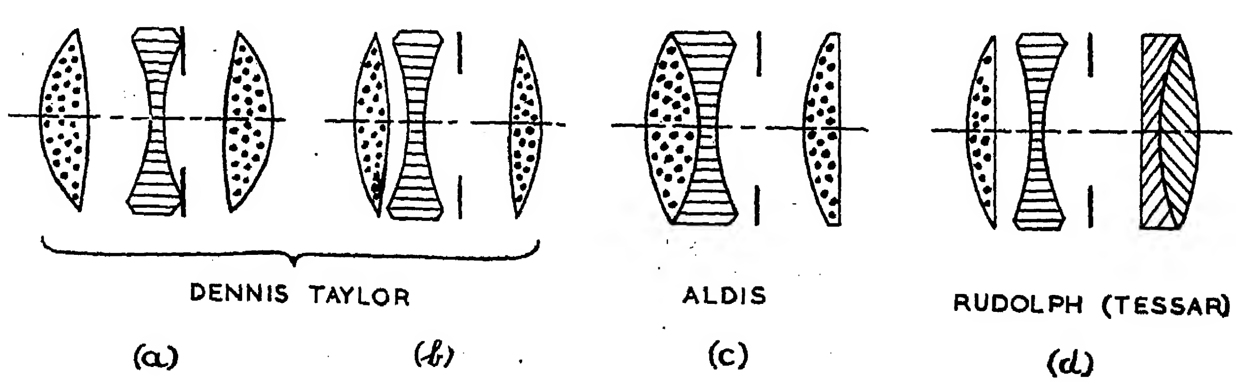


Fig. 60 (Diagrammatic only).

towards the Petzval sum tends more nearly to balance out the positive contribution given by the convergent lenses, thus giving a smaller total Petzval sum than would otherwise be the case.

Some of the forms of the "Cooke" lens are shown diagrammatically in Fig. 60 (*a*) and (*b*) and in modified form Fig. 60 (*c*) and (*d*); but the general principle of utilizing a negative component with suitable air-spacing between the lenses in order to reduce the Petzval sum, will be evident.

N.B.—The original Taylor lens had the three components (Fig. 60a) separately achromatized, but it has since been found possible to give sufficient colour correction by using single lenses made from the appropriate glasses.

For the purpose of the designing work here we will choose the form shown in Fig. 61 (*a*), for with this type there is a greater drop in height of the rays incident on the concave lens, and also it is easier to illustrate the principles of the design by treating the lens system as being symmetrical about a dotted line drawn through the centre of the concave lens (see Fig. 59).

The specification we will ask for, is that the equivalent focal length of the lens shall be ten inches, and that it should cover a half-plate (i.e., 8 inches across the diagonal or $H'_k = 4$ inches). An aperture of two inches will be used at first (i.e., an F/ratio of 5), but will be increased later if the aberrations will permit this.

The following glasses will be used :—

	N_D	$N_{G'}$	V	$\delta N = (N_{G'} - N_D)$	V/N
Dense barium crown ..	1·6123	1·6256	58·5	0·01330	36·3
Light silicate flint	1·5673	1·5843	43·8	0·01700	27·9

We may commence the design by treating the lens system as a symmetrical type, with the front component to the left of the dotted line (see Fig. 59) and therefore the curvatures (giving achromatism) of the lens *a* and the half-lens *b* may be calculated from

$$\mathcal{R}_a = \frac{1}{f'} \cdot \frac{1}{\delta N_a} \cdot \frac{1}{V_a - \left(\frac{l'_a}{l_b} \cdot V_b\right)} \quad \text{and}$$

$$\mathcal{R}_a = -\left(\frac{l'_a}{l_b}\right)^2 \cdot \frac{1}{f'} \cdot \frac{1}{\delta N_b} \cdot \frac{1}{V_a - \left(\frac{l'_a}{l_b} \cdot V_b\right)}$$

where $\frac{l'_a}{l_b}$ is in effect (from the lettering stated) a measure of the separation of the lenses. [See Conrady's *Applied Optics*, pp. 175–182.]

Putting in the appropriate focal length and separation indicated by the Petzval sum formula for separated lenses we find that $\mathcal{R}_a = +0 \cdot 2557$ and $\mathcal{R}_b = -0 \cdot 2000$.

From these curvatures, we can now prepare various shapes of the lenses, and if we are going to assume the system as being a symmetrical one (in the first instance) this automatically fixes the central diverging lens as being equi-concave and therefore it will have radii of $r_3 = -5 \cdot 000$ and $r_4 = +5 \cdot 000$ inches. The two outer converging lenses can, however, be changed in shape as desired, and the most suitable " bending " chosen which will give freedom from spherical aberration for the complete lens system. Let us choose (quite arbitrarily) as one shape of the front and rear lens, a plano-convex lens ; then from $\mathcal{R}_a = R_1 - R_2$ we find $r_1 = 3 \cdot 911$ inches and the specification becomes :—

$r_1 = +3 \cdot 911$	$r_3 = -5 \cdot 000$	$r_5 = \infty$
$r_2 = \infty$	$r_4 = +5 \cdot 000$	$r_6 = -3 \cdot 911$

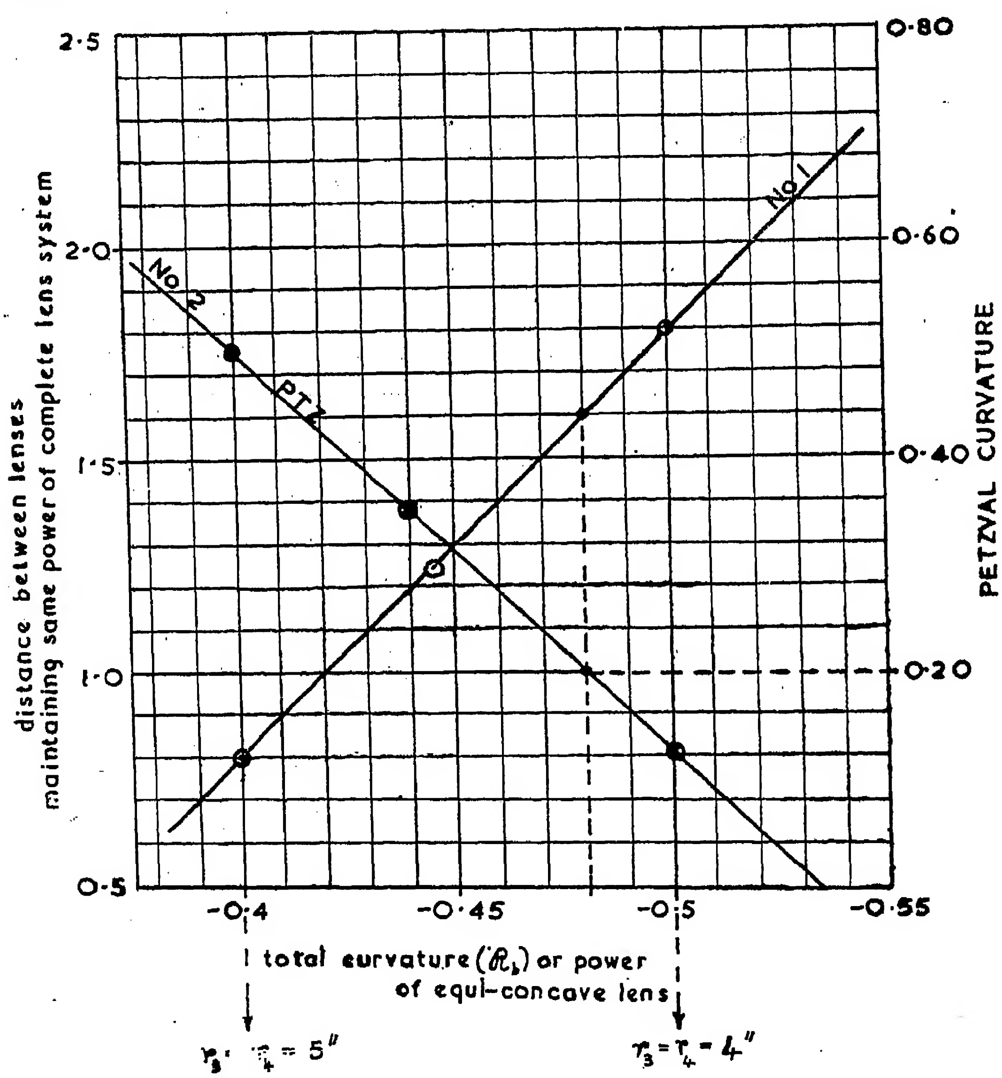


Fig. 61. Relationship between Petzval curvature and various powers of the central equi-concave lens, with the corresponding necessary separation of the lenses which maintains the required power (or focal length) of the complete system.

From scale drawings, a suitable axial thickness of the convex and concave lenses will be found to be 0·5 inch and 0·25 inch respectively.

As yet the separation of the lenses is not known, and we must proceed to find this. Let us first consider the Petzval curvature and see how it may be varied by changing the radii of the equi-concave central lens but simultaneously arranging the corresponding spacing of the lenses so that the same focal length of the complete lens system is maintained. For example, if the power and the shape of the two external lenses is kept constant, and the equi-concave lens is then varied by making $r_3 = r_4$ equal in turn to 5 inches, 4·5 inches and 4 inches, we shall find the corresponding separation of the lenses is 1·8, 1·25 and 0·8 inches respectively; in order to secure the same equivalent focal length of the complete lens system in each case. The foregoing figures can be determined by ray-tracing experiments.

The graph of the above values is plotted on curve No. 1 of Fig. 61 and gives a useful piece of information, for it tells us the necessary separation of the lenses to employ when the power of the central concave lens is varied. Further than this, the Petzval curvature [calculated from $\frac{1}{r_{ptz}} = \Sigma\left(\frac{N' - N}{N' . N . r}\right)$ for each of the three cases mentioned above] may also be plotted on the same graph and the way in which it changes with the power of the central divergent lens, can also be found. Curve No. 2 of Fig. 61 shows this relationship.

It will be noticed from these graphs that it is possible to arrange the design so that the Petzval sum is zero—the first time it has been possible to secure this condition with any of the photographic lenses dealt with in this chapter—in fact the Petzval surface could be made to bend *away* from the lens as indicated by the fact that curve No. 2 passes below the abscissa. It should be observed, however, that both the separation of the lenses and the power of the central diverging lens become excessive.

For this reason, it is better not to aim at a complete zero value for the Petzval sum, but rather to choose a value for r_{ptz} of (say) 40 or 50 inches.

[N.B.—Sometimes "anastigmats" may be classified into categories depending on whether the ratio r_{ptz}/E.F.L. is high or low; a value of 4 or 5 (as in the case here) is considered quite good.]

Let us therefore choose from Fig. 61, a value of $R_{ptz} = 0 \cdot 020$ (i.e., $r_{ptz} = 50$ inches); and by following the dotted lines on the graph, a total curvature or power of the concave lens will be found to be $-0 \cdot 48$ whilst the separation of the lenses will be 1·60 inches.

These values, together with the power of the two external lenses already calculated (namely $R_a = +0 \cdot 2557 = R_c$) enables specifications to be drawn up for various shapes of the lenses.

Suppose, for example, it is decided to keep the central diverging lens as an equi-concave one, then by "bending" the two outer ones, we might have the following specifications (see Fig. 62) :—

(1)		(2)		(3)	
$r_1 = +3\cdot750''$	$d_1' = 0\cdot50$	$r_1 = +4\cdot000$	$d_1' = 0\cdot50$	$r_1 = +4\cdot250$	$d_1' = 0\cdot50$
$r_2 = +90\cdot909''$		$r_2 = -175\cdot44$		$r_2 = -49\cdot020$	
Air space	$d_2' = 1\cdot60''$	Air space	$d_2' = 1\cdot60$	Air space	$d_2' = 1\cdot60$
$r_3 = -4\cdot167$	$d_3' = 0\cdot25$	$r_3 = -4\cdot167$	$d_3' = 0\cdot25$	$r_3 = -4\cdot167$	$d_3' = 0\cdot25$
$r_4 = +4\cdot167$		$r_4 = +4\cdot167$		$r_4 = +4\cdot167$	
Air space	$d_4' = 1\cdot60$	Air space	$d_4' = 1\cdot60$	Air space	$d_4' = 1\cdot60$
$r_5 = -90\cdot909$	$d_5' = 0\cdot50$	$r_5 = +175\cdot44$	$d_5' = 0\cdot50$	$r_5 = +49\cdot020$	$d_5' = 0\cdot50$
$r_6 = -3\cdot750$		$r_6 = -4\cdot000$		$r_6 = -4\cdot250$	

The spherical aberration must now be determined (trigonometrically) for each of the foregoing three lens systems by tracing a marginal and paraxial ray (with $Y = 1\cdot00 = y$) in G′ light through each of them. The spherical

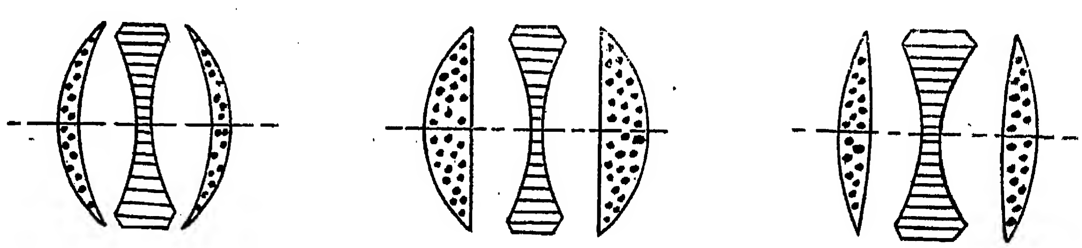


Fig. 62.

aberration values obtained should then be plotted as ordinate against the curvature R_1 of the first surface of the front lens, when it will be found from the graph that for zero spherical aberration (or nearly so) R_1 will equal $+0\cdot2463$, and hence $r_1 = +4\cdot060$ inches. Thus the specification for the complete lens system when corrected for spherical aberration will be :—

$r_1 = +4\cdot060$	
$r_2 = -106\cdot380$	$d_1' = 0\cdot50$
$r_3 = -4\cdot167$	$d_2' = 1\cdot60$ (air-space)
$r_4 = +4\cdot167$	$d_3' = 0\cdot25$
$r_5 = +106\cdot380$	$d_4' = 1\cdot60$ (air-space)
$r_6 = -4\cdot060$	$d_5' = 0\cdot50$

CALCULATION NO. 33

Surface	1*st*	2*nd*	3*rd*	4*th*	5*th*	6*th*
L $-r$		9·9272 106·380	4·243 4·167	18·5466 4·167	−19·2272 −106·380	−35·765 4·060
$(L-r)$		116·3072	8·410	14·3796	−125·6072	−31·705
log sin U + log $(L-r)$	$Y=1·00$	8·98504 2·06561	9·21158 0·92480	8·57550 1·15775	8·59475*n* 2·09901*n*	8·33155*n* 1·50113*n*
log $(L-r)$ sin U − log r	0·00000 0·60853	1·05065 2·02686*n*	0·13638 0·61982*n*	9·73325 0·61982	0·69376 2·02686	9·83268 0·60853*n*
log sin I + log $\left(\frac{N}{N'}\right)$	9·39147 −0·21101	9·02379*n* 0·21101	9·51656*n* −0·19984	9·11343 0·19984	8·66690 −0·21101	9·22415*n* 0·21101
log sin I' + log r	9·18046 0·60853	9·23480*n* 2·02686*n*	9·31672*n* 0·61982*n*	9·31327 0·61982	8·45589 2·02686	9·43516*n* 0·60853*n*
log r . sin I' − log sin U'	9·78899 8·98504	1·26166 9·21158	9·93654 8·57550	9·93309 8·59475*n*	0·48275 8·33155*n*	0·04369 8·93421
log $(L'-r)$	0·80395	2·05008	1·36104	1·33834*n*	2·15120*n*	1.10948
U $+I$	0– 0– 0 14–15–32	5–32–39 − 6– 3–49	9–22– 4 −19–10–44	2– 9–23 7–27–39	− 2–15–15 2–39–43	− 1–13–46 − 9–38–44
$U+I$ $-I'$	14–15–32 8–42–53	−0–31–10 +9–53–14	−9–48–40 +11–58– 3	9–37– 2 11–52–17	0–24–28 −1–38–14	−10–52–30 15–48–19
U'	5–32–39	9–22– 4	2– 9–23	−2–15–15	−1–13–46	4–55–49
$L'-r$ $+r$	6.3672 4·060	112.223 −106·380	22.9636 −4·167	−21·7942 4·167	−141·645 +106·380	12·8671 −4·060
L' $-d'$	10·4272 0·50	5·843 1·60	18·7966 0·25	−17·6272 1·60	−35·265 0·50	8·8071
new L	9·9272	4·243	18·5466	−19·2272	−35·765	

CALCULATION NO. 34

	r_1	r_2	r_3	r_4	r_5	r_6
N' $-N$	1·6123 1·0000	1·0000 1·6123	1·5673 1·0000	1·0000 1·5673	1·6123 1·0000	1·0000 1·6123
$(N'-N)$	0·6123	−0·6123	0·5673	−0·5673	0·6123	−0·6123
log $(N'-N)$ + colog N' + colog N + colog r	9·7870 9·7926 0·0000 9·3915	9·7870*n* 0·0000 9·7926 7·9731*n*	9·7538 9·8048 0·0000 9·3802*n*	9·7538*n* 0·0000 9·8048 9·3802	9·7870 9·7926 0·0000 7·9731	9·7870*n* 0·0000 9·7926 9·3915*n*
log sum	8·9711	7·5427	8·9388*n*	8·9388*n*	7·5427	8·9711
$\left(\frac{N'-N}{N'.N.r}\right)$	0·0936	0·0035	−0·0868	−0·0868	0·0035	0·0936

$\therefore \quad \frac{1}{r_{ptz}} = 0·0206$ $\qquad r_{ptz} = 48·54$ inches.

As a matter of interest the trace for the extreme marginal ray of the above specification is given in Calculation No. 33 in order to give a mental picture as to the inclination of the rays to the various surfaces and to the axis as the light passes right through the lens. The numerical work for the Petzval sum is also shown, this being indicated in Calculation No. 34.

We will now pass on to test the astigmatism at various angular fields. This involves first of all the tracing of principal rays (commencing at the centre of the concave lens) at such angles so that the final emergent rays make angles with the axis of (say) 22 degrees, 16 degrees and 9 degrees. In this particular example (which is purely an illustrative one) the principal rays have been traced from the centre of the concave lens at angles of 22 degrees and 12 degrees to the axis only (right to left and left to right) and it will be found that they emerge from the last surface of the lens system at closely similar angles, namely

U_{pr}	$-22°$	$-12°$
U'_{pr6}	$-22°$–31′–47″	$-12°$–57′–37″
L'_{pr6}	$-3 \cdot 5182$	$-3 \cdot 0267$

The astigmatism is then calculated surface by surface as already

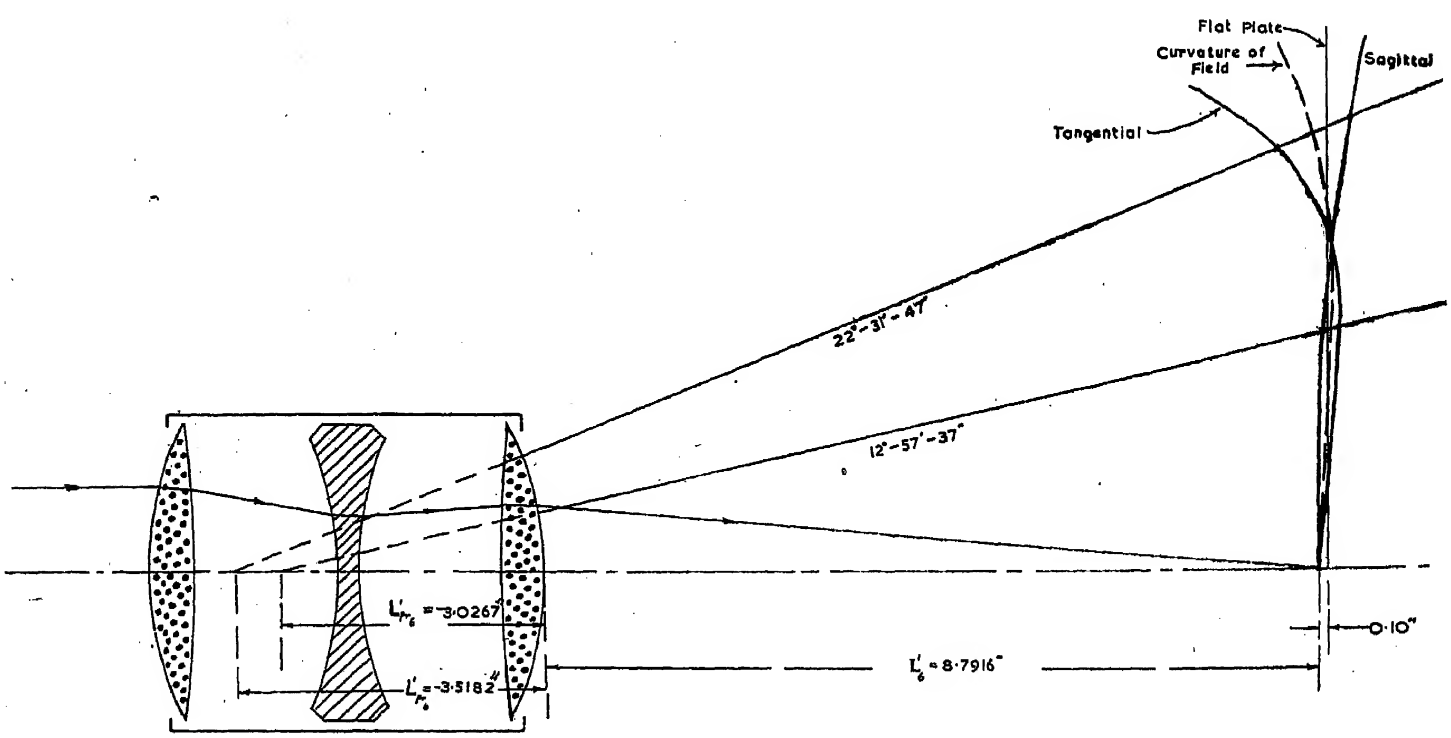


Fig. 63.

indicated by the method given on page 90 when the following figures will be found :—

Surface		(1)	(2)	(3)	(4)	(5)	(6)
22°	X_{pr}	+0·2352	−0·0077	−0·0003	+0·0003	+0·0077	−0·2352
inclination	D'_{pr}	0·2761	2·0123	0·2703	2·0123	0·2761	—
	s'_{pr}	10·5419	6·0580	18·3498	−13·6593	−28·4741	10·2395
	t'_{pr}	10·5312	4·3866	13·0991	−11·2380	−34·4072	9·2884
12°	X_{pr}	0·0579	−0·0016	−0·0001	+0·0001	+0·0016	−0·0579
inclination	D'_{pr}	0·4498	1·6986	0·2558	1·6986	0·4498	—
	s'_{pr}	10·5395	5·9805	18·0240	−17·7084	−35·7824	9·1432
	t'_{pr}	10·5264	5·5189	17·5817	−15·1544	−33·4858	9·3678

The final l'_s and l'_t values will be found to be :—

At 22 degree inclination, $l'_s = 9 \cdot 2228$; and $l'_t = 8 \cdot 3443$.

At 13 degree inclination, $l'_s = 8 \cdot 8524$; and $l'_t = 9 \cdot 0712$.

giving an astigmatism value $(l'_s - l'_t)$ equal to $+\,0 \cdot 8785$ inches and $-\,0 \cdot 2188$ inches respectively.

These figures are then drawn out carefully to scale as shown in Fig. 63 by the methods already described earlier in this chapter and the astigmatic surfaces drawn in. One of the more important things to notice is that the astigmatic fields remain fairly close to one another up to about H'_k equal to 4 inches (i.e., an angular field of approximately 17 degrees) when they *cross over* and then proceed to separate rather rapidly. Obviously it would be advisable to calculate the astigmatism at other inclinations such as at 17 and 7 degrees, in order to get a better idea as to the shape of the true curvature of field ; students will find it instructive to do this.

It will be noticed also that the astigmatic fields slope *back* slightly from the perpendicular when passing from the axial focus out to where (say) H'_k equals 4 inches. This is explained by the fact that the Petzval curvature chosen was distinctly large (namely about 50 inches), and as there is some residual astigmatism still present it follows that the usual three-to-one ratio of the T and S distances from the Petzval surface will hold, and therefore the fields must be curved away from the lens with respect to a vertical line drawn from the axial focus. Hence by making the Petzval curvature slightly smaller (e.g., 40 or 30 inches radius) it will be possible to secure the condition such that the true curvature of field lies almost on a flat plate perpendicular to the axis. This is a matter for experiment in the design. (By referring to the graphs of Fig. 61 the corresponding power of the central lens and the air-space separation can be obtained for the new Petzval curvature chosen.) The rapid separation of the sagittal and tangential fields beyond $H'_k = 4$ inches, and the breakdown of the usual 3 to 1 distance ratio from the Petzval surface is

due to the effects of higher order aberrations coming into play at the wider angular field.

It will be obvious that this example given here does not represent the best possible solution, but it does indicate the lines along which the systematic design of such a lens may be conducted, the refinement in the designing process only being obtained by continued experience.

If one should want to make the best use of the result given on Fig. 63 it might be desirable to place the flat plate in a position about 1/10th of an inch to the right of the axial focus, when the definition (although rather poor) would be more uniform over the whole area of the plate. The discs of least confusion in such a case would have diameters of 0·088, 0·000, 0·022, 0·012 and 0·017 inches at inclinations of 22°, 17°, 13°, 7° and on the axis respectively.

It would, however, be better to improve the design, so that the curvature of field lies on a flat plate up to approximately 17 degrees, when the other aberrations such as the distortion and the coma should also be tested, including the chromatic aberration (which was only dealt with in the opening equations for giving the curvature values) and which should be corrected if necessary by adjustment of the last radius of the lens system.

Chapter V

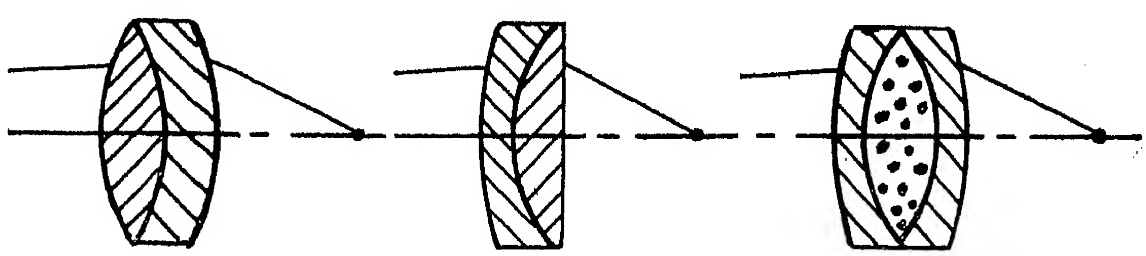
THE DESIGN OF MICROSCOPE OBJECTIVES

IN this chapter the design of a low power, medium power and a high power microscope objective will be illustrated. As in the previous chapters no finality in any one design is attempted, but merely the outline of approach is given for carrying out the design in a systematic way.

It should be emphasized at the outset that owing to the small radius of curvature of the lens surfaces utilized in microscope objectives some of the analytical methods previously used for the partial solving of the problem do not give particularly reliable results, and greater use has to be made of ray-tracing work. Although this may make the total time taken for a design longer, *systematic* methods can still be employed which thus prevent the solving of the problem from becoming a wearisome hunt for a solution which otherwise always appears a long way off.

Low-Power Objectives

The three types of lenses for low power work (i.e., those having a focal length of between 75 mm. and 25 mm., and an upper limit in Numerical Aperture of about 0·15) may be simple cemented combinations such as those depicted in Fig. 64 (*a*), (*b*) and (*c*).



(a) (b) (c)

Fig. 64.

Types (*a*) and (*b*) cannot be fully corrected for chromatic aberration, spherical aberration and fulfilment of the sine condition simultaneously* ; but type (*c*), having an additional radius, does allow of this and moreover also admits of a somewhat higher numerical aperture.

As a numerical example of a two-lens cemented low power objective, let us ask for the following specification :—

Equivalent focal length to be 25 mm.

Numerical Aperture to be 0·15.

* Except by the choice of somewhat special pairs of glasses.

Optical Tube Length to be 160 mm.
Primary Magnification = 6·4 ×.

The choice of glass types to use may present some difficulty unless experience has been previously obtained, but it will be found that the best results regarding a low *OSC* value will be secured by using a Barium Crown glass of $N_D = 1{\cdot}57$ to $1{\cdot}58$ and of high V value combined with an ordinary Dense Flint glass. For example, the two following types might serve quite well :—

			Mean Dispersion	Partial Dispersions	
	N_D	V	$(N_F - N_C)$	$(N_D - N_C)$	$(N_F - N_D)$
Medium barium crown ..	1·57220	57·7	0·00990	0·00299	0·00691
Dense Flint	1·62046	36·1	0·01718	0·00511	0·01207

We commence the design by collecting and calculating some of the initial data. This will be aided by making a drawing (see Fig. 65). It is more convenient in practice to compute microscope objectives for the reverse of

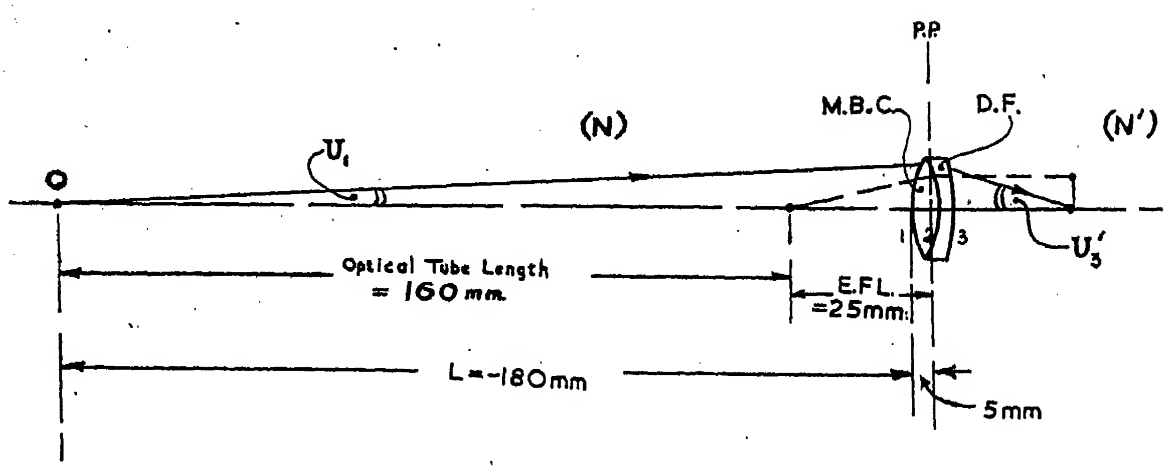


Fig. 65.

the actually used direction ; for by doing this we can (i) make sure of the desired optical tube length and (ii) make final adjustments more easily by alteration of the front surface or in the case of higher power lenses by alteration of the front lens.

Thus, referring to Fig. 65, we start with our object at O and with an initial ray proceeding towards the lens under an angle U_1. After passing through the lens the ray will converge towards the axis under the angle U_3' (i.e., the numerical aperture angle). The optical tube-length (that is, the distance between the upper focal plane of the objective and what is usually the image plane at O) is indicated on the diagram and is taken as the recognized standard of 160 mm. Allowing for a distance of, say, 5 mm. from the principal plane

to the pole of surface No. 1, this gives an initial object distance L as 180 mm.; and in accordance with the sign convention adopted here it will have a negative sign.

In order to obtain the initial angle U_1 we make use of the Optical Sine Relation (viz. $N \, . \, h \, . \sin U = N' \, . \, h' \, . \sin U'$) and write

$$\frac{N' \, . \sin U_3'}{N \, . \sin U_1} = \frac{h}{h'} = M \; ; \quad \text{or} \quad \frac{0 \cdot 15}{\sin U_1} = -6 \cdot 4$$

so that, $\sin U_1 = -0 \cdot 0234$ and therefore $U_1 = -1°\text{–}20'\text{–}0''$.

We now find the curvature of each component which when combined will give achromatism from :—

$$R_a = \frac{1}{\text{E.F.L.}} \cdot \frac{1}{(V_a - V_b)} \cdot \frac{1}{\delta N_a} = \frac{1}{25 \times 21 \cdot 6 \times 0 \cdot 00990} = +\,0 \cdot 1871$$

$$R_b = \frac{1}{\text{E.F.L.}} \cdot \frac{1}{(V_b - V_a)} \cdot \frac{1}{\delta N_b} = \frac{1}{25 \times -21 \cdot 6 \times 0 \cdot 01718} = -\,0 \cdot 1077$$

Having found these values for R_a and R_b we would have in most cases applied the analytical G-sum formulæ (see page 36) for securing the spherical aberration for different shapes of the complete lens system but this method does not give sufficiently reliable results when dealing with the small radii employed in microscope objectives.

It is necessary therefore to take three (or sometimes four) separate "bendings" of the lens and to trace trigonometrically three rays (namely, marginal, paraxial and zonal rays) through each shape, correcting the last surface for achromatism in each case.

Let us take three shapes given, for instance, by putting R_2 (the contact curvature) equal in turn to $-0 \cdot 122$, $-0 \cdot 132$ and $-0 \cdot 142$; and from $R_a = R_1 - R_2$ and $R_b = R_2 - R_3$ we find the following radii :—

1st Bending	*2nd Bending*	*3rd Bending*
$r_1 = +\,15 \cdot 36$ mm.	$r_1 = +\,18 \cdot 15$ mm.	$r_1 = +\,22 \cdot 17$ mm.
$r_2 = -\;\;8 \cdot 20$ mm.	$r_2 = -\;\;7 \cdot 58$ mm.	$r_2 = -\;\;7 \cdot 04$ mm.
$r_3 = -\,69 \cdot 93$ mm.	$r_3 = -\,41 \cdot 15$ mm.	$r_3 = -\,29 \cdot 16$ mm.

Radius r_3 is uncorrected in each case.

The axial thicknesses $d_1' = 2 \cdot 50$ mm. and $d_2' = 1 \cdot 50$ mm. of the lenses are obtained by making a scale drawing (ten times full size) of the above shapes. The necessary semi-diameter of the objective will be found by multiplying the tangent of U_1 (namely $1°\text{–}20'\text{–}0''$) by the length 180 mm. (see Fig. 66) which will be found to be $4 \cdot 2$ mm.; so that the full diameter, allowing for mounting the lens, may be taken as 9 mm.

Taking No. 1 bending first, the ray-tracing is then commenced (in N light) with the following data :—

Marginal ray	*Paraxial ray*	*Zonal ray*
$L = -180$ mm.	$l = -180$ mm.	$L_z = -180$ mm.
$U_M = -1°–20'–0''$	$u =$ nominal U_M	$U_z = -0°–57'–0''$ (i.e., $\sqrt{0 \cdot 5} \times$ S.A.)

	Crown	*Flint*
N_y	1·57406	1·62369

N.B.—It is nearly always advisable when computing microscope objectives to trace a zonal ray (at $\sqrt{0 \cdot 5}$ of the semi-aperture) as the steeply curved lens surfaces often produce large amounts of zonal spherical aberration even if the marginal spherical aberration is corrected.

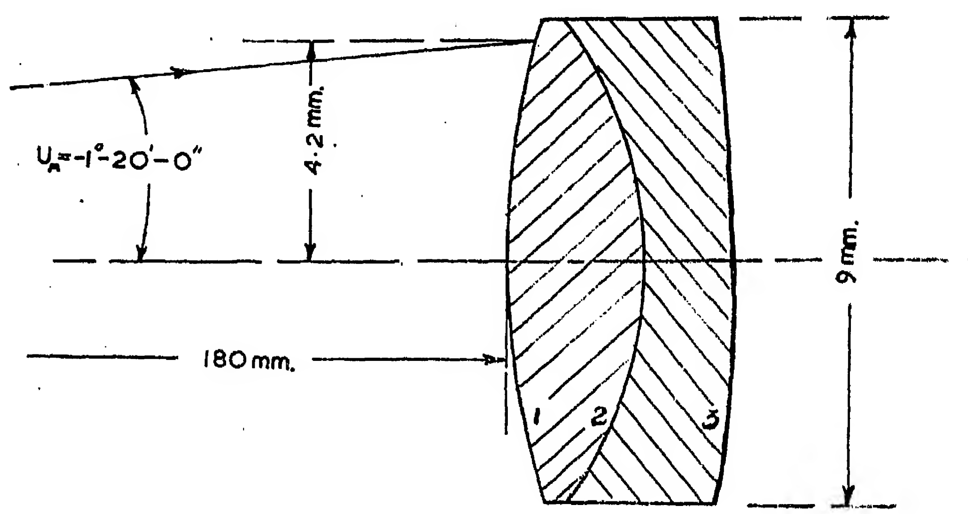


Fig. 66.

Having traced the rays through the second surface we can now introduce another method (other than Chr. I used in chapter II) for securing the correct last radius giving complete achromatism. The method is known as the $(d - D)$ method and gives a more exact result than is possible with the Chr. I method when bold curvatures are employed. The derivation of the necessary formulæ and the application of these to the method are shown at the end of this section.

Assuming, therefore, that each of the three lens-shapes has been achromatized by this method, the marginal and paraxial ray tracing is then continued on through the last surface when the following results will be obtained :

	1st Bending	*2nd Bending*	*3rd Bending*
	$R_2 = -0 \cdot 122$	$R_2 = -0 \cdot 132$	$R_2 = -0 \cdot 142$
Marg. Sph. Ab.	$-0 \cdot 064$ mm.	$-0 \cdot 142$ mm.	$-0 \cdot 070$ mm.
Zonal. Sph. Ab.	$+0 \cdot 058$ mm.	$+0 \cdot 035$ mm.	$+0 \cdot 097$ mm.
OSC	$+0 \cdot 0060$	$-0 \cdot 0002$	$-0 \cdot 0063$

To face page 145.

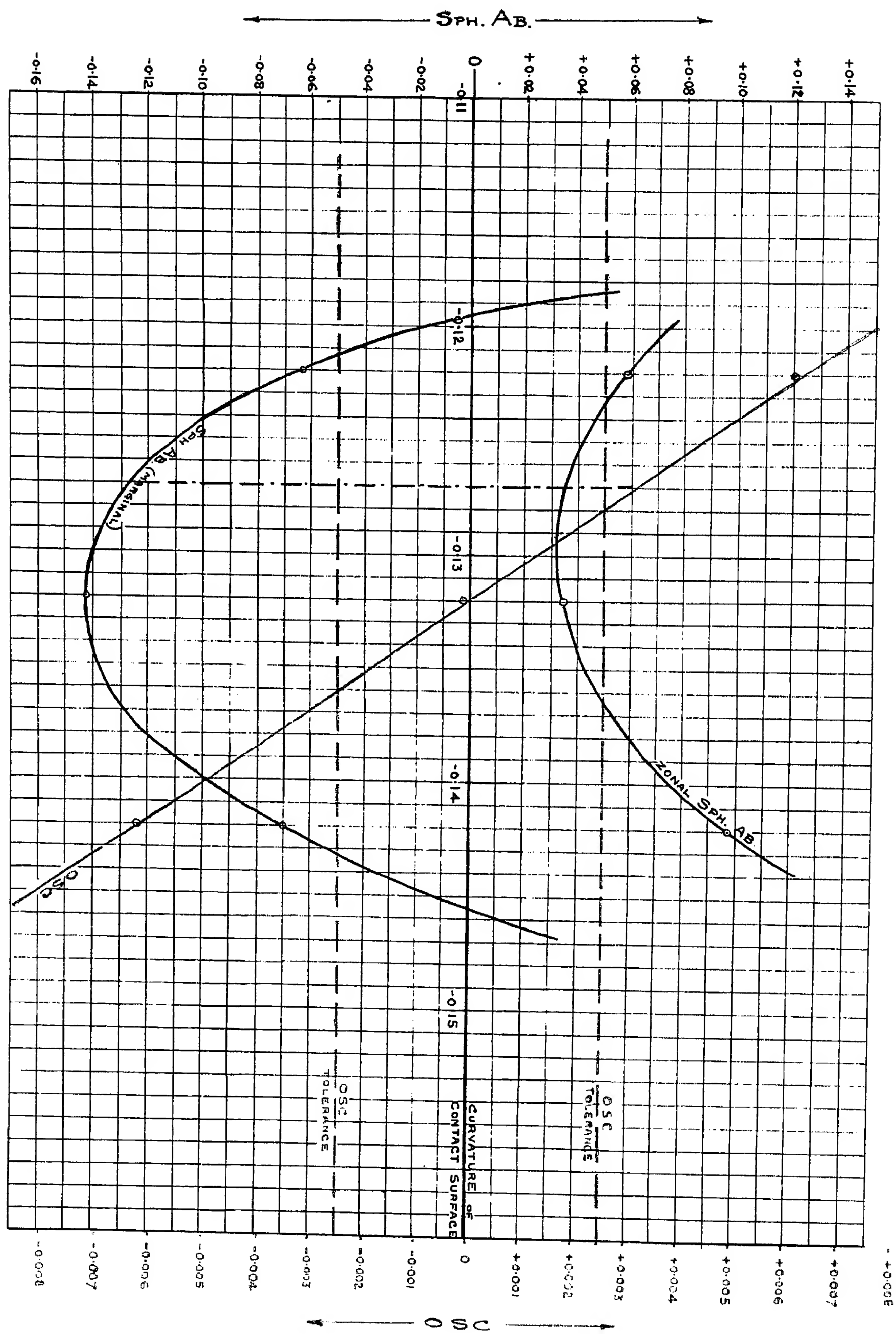


Fig. 67.

The last line of the above table is obtained from the usual " Offence against the Sine Condition " formula, namely

$$OSC = 1 - \left(\frac{u'_k \cdot l'_k}{\sin U'_k \cdot L_k}\right)$$

The foregoing values are then plotted with the curvature of the contact surface R_2 as abscissa and the spherical aberration and the OSC as ordinates, when the graphs will appear as in Fig. 67. They can then be studied, and several interesting points may be noted.

If the respective tolerances are drawn in over the aberration curves, it will be seen firstly that for zero amounts of marginal spherical aberration (namely at $R_2 = -0 \cdot 1195$ and $-0 \cdot 1455$) the OSC value is $+0 \cdot 0076$ and $-0 \cdot 0085$ respectively and therefore exceeds the tolerance by slightly more than three times in either case.

Secondly, if we choose the shape (with $R_2 = -0 \cdot 1318$) which gives a zero OSC value, then the marginal spherical aberration is well outside its permissible tolerance.

Hence, one must choose some compromise solution and probably the most suitable for this lens (under the given set of conditions) would be when R_2 was in the region of $-0 \cdot 128$ or $-0 \cdot 127$. The writer has chosen the latter value for R_2 for the final specification.

This gives :—

$$r_1 = +16 \cdot 64 \text{ mm.} \qquad d'_1 = 2 \cdot 50 \text{ mm.}$$
$$r_2 = -\ 7 \cdot 87 \text{ mm.} \qquad d'_2 = 1 \cdot 50 \text{ mm.}$$
$$r_3 = -41 \cdot 41 \text{ mm.}$$

r_3 has been corrected by $(d - D)$ method—see page 149.

When tested trigonometrically, the following aberrations will be found.

Chromatic aberration $(L'_C - L'_F)$ $= +0 \cdot 003$ mm.
Marginal spherical aberration $(l'_y - L'_{yM}) = -0 \cdot 123$ mm.
Zonal spherical aberration $(l'_y - L'_{yZ}) = +0 \cdot 035$ mm.
Offence against the Sine Condition $(OSC) = +0 \cdot 0029$

If the tolerances are then worked out, it will be seen that the aberrations (with the exception of the chromatic) are all slightly above the permissible tolerances; and therefore such a lens would not be quite good enough in performance for the working numerical aperture of $0 \cdot 15$. It would really be better to use a triple lens as in Fig. 64c or a Lister type objective as described

in the next section. Nevertheless it is instructive to see what can be done with a two-lens cemented objective.

If, however, the full N.A. were reduced to that given by the zonal ray (i.e., $U_3' = 6°–20'–57''$) namely N.A. $= 0 \cdot 11$, the lens would probably be quite satisfactory; but this would mean a certain amount of re-designing. For the OSC values must be re-calculated from the figures of the now new marginal ray and a new zonal ray must be traced with $U_2 = -0°–40'–0''$; a fresh set of graphs would have to be drawn and the newly calculated tolerances put on, and a possible modification in the choice of the best solution made. It will, however, be interesting to carry this out if one is keen to obtain additional experience.

The $(d - D)$ method for effecting Chromatic Correction

This method admits a useful solution of the problem of determining the value of the last radius of a lens-combination which will make the system achromatic. It is rather better than the Chr. 1 and Chr. 3 methods (given earlier) in the cases of microscope and photographic lenses where pronounced curvatures are involved.

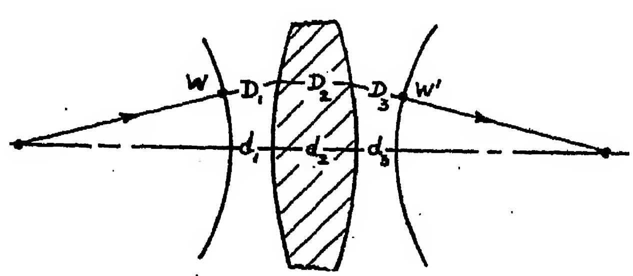


Fig. 68.

The method is based on the principle of equal optical paths (see Fig. 68). The path-length between the initial wavefront W and the final wavefront W′ is equal to $d_1N + d_2N + d_3N$ for the axial ray, and $D_1N + D_2N + D_3N$ for the marginal ray;

or $\Sigma d : N$ for pathlength along the axis

and $\Sigma D . N$ for pathlength along the marginal ray.

These two pathlengths must be equal, so that

$$\Sigma (d - D)N = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For the chromatic condition, if δN is taken as the small increase in the refractive index corresponding to a small decrease in wavelength (generally from C to F) we shall have:

$$D_1 + N \, . \, D_2 + D_3 = d_1 + N \, . \, d_2 + d_3$$

and $$D_1 + (N + \delta N) D_2 + D_3 = d_1 + (N + \delta N) d_2 + d_3$$

i.e., $$\Sigma\,(d - D)(N + \delta N) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

Hence, the difference of the foregoing sums (1) and (2) namely $\Sigma\,(d - D)\delta N = 0$ is the expression which represents the chromatic condition.

Let us assume that a lens system of k surfaces has been calculated up to and including the *last-but-one* surface, and the $(d - D)\delta N$ sum has also been determined up to that point. The problem then is to determine the value of the last radius r_k which will give the desired chromatic correction. Usually the latter will be that the $(d - D)\delta N$ sum for the whole system shall be zero but in order to allow for cases of *over* or *under* correction we will make the solution more general. [Let the symbol for the amount of over or under-correction be Chr.]

Then, the whole sum is made up of that already determined for the first lenses,* and of $(d_k - D_k)\delta N_k$, hence we have the condition

$$\Sigma_{(2)}^{(k-1)} (d - D)\delta N + (d_k - D_k)\delta N_k = \text{Chr.} \quad . \quad . \quad . \qquad (1)$$

In this formula (1) the only unknown quantity is D_k and by transposition we find for it the value

$$D_k = d_k + \frac{\Sigma_{2}^{(k-1)} (d - D)\delta N - \text{Chr}}{\delta N_k} \quad . \quad . \quad . \quad . \qquad (2)$$

By an earlier equation (see page 20) $D_k = (d_k + X_k - X_{(k-1)}) \sec U'_{(k-1)}$ in which X_k is the only unknown quantity, and is found from

$$X_k = D_k \cos U'_{k-1} + X_{(k-1)} - d_k \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Also from an earlier equation

$$Y_k = Y_{(k-1)} - D_k \sin U'_{(k-1)} \quad . \quad . \quad . \quad . \qquad (4)$$

And by the spherometer formula,

$$r_k = \frac{Y_k^2}{2X_k} + \frac{X_k}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

Let us apply the foregoing to a two-lens system (such as the microscope objective we have been dealing with) and give the above formulæ the necessary suffixes.

* The sum begins with the *second* surface because the $(d - D)$ values apply to the individual *lenses*, and the first D is therefore that in front of the second surface.

Referring to Fig. 69, X_1 and X_2 are first calculated from

$$X = \frac{PA^2}{2r} \text{ if } PA\text{-check has been used in the ray tracing}$$

or

$$X = 2r \,.\, \sin^2\left(\frac{U+I}{2}\right) \text{ if } PA\text{-check has not been used.}$$

With axial thickness d_1' known ; valuate $(d_1' + X_2 - X_1)$.

Then $D_1' = (d_1' + X_2 - X_1) \sec U_1'$

$(d_1' - D_1')$ can then be obtained and multiplying this by $\delta N_1'$ we get

$$(d_1' - D_1')\delta N_1'$$

From formula (2) above :

$$D_2' = d_2' + \frac{(d_1' - D_1')\delta N_1' - \text{Chr}}{\delta N_2}$$

N.B.—Chr is generally zero. δN_2 is the mean dispersion for the second glass.

Utilizing D_2' and the convergence angle U_2' from the ray-tracing we obtain from formula (3)

$$X_3 = D_2' \,.\, \cos U_2' + X_2 - d_2'$$

And $Y_3 = Y_2 - D_2' \,.\, \sin U_2'$

N.B.—$Y_2 = r_2 \,.\, \sin(U_2 + I_2)$

Finally $$r_3 = \frac{(Y_3)^2}{2X_3} + \frac{X_3}{2}$$

As a numerical illustration of the use of these equations, we will calculate the last radius for the microscope objective (best solution) given on page 145.

The following values obtained from the ray-tracing through surfaces 1 and 2 are required :—

	$r_1 = +16 \cdot 64$ mm.	$r_2 = -7 \cdot 87$ mm.
	1st Surface	*2nd Surface*
PA	4·23653	4·30328
U'	4°–34′–0″	3°–17′–26″

$U_2 = 4°–34′–0″$ and $I_2 = -36°–18′–0″$.

and the arrangement of the calculation is shown in calculation No. 35.

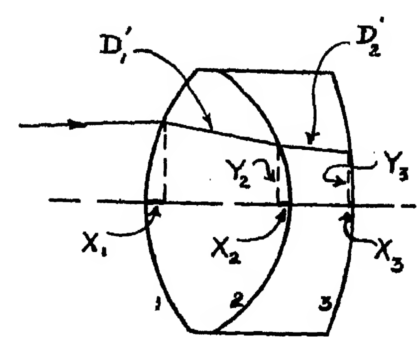


Fig. 69.

CALCULATION No. 35

	X_1	X_2
$2 \log PA$ $- \log 2r$	1·25402 1·52218	1·26760 1·19700n
$\log X$	9·73184	0·07060n
X	+0·53931	−1·17652

d_1' $+ X_2$	2·50000 −1·17652
$d_1' + X_2$ $- X_1$	1·32348 0·53931
$(d_1' + X_2 - X_1)$	0·78417
$\log (d_1' + X_2 - X_1)$ $+ \log \sec U_1'$	9·89441 0·00138
$\log D_1'$	9·89579
D_1'	0·78666
d_1' $- D_1'$	2·50000 0·78666
$(d_1' - D_1')$	1·71334
$\log (d_1' - D_1')$ $+ \log \delta N_1'$	0·23384 7·99564
$\log (d_1' - D_1')\delta N_1'$ $- \log \delta N_2'$	8·22948 8·23502
log 2nd term	9·99446
antilog 2nd term $+ d_2'$	0·98732 1·50000
D_2'	2·48732

$\log D_2'$ $+ \log \cos U_2'$	0·39573 9·99928
$\log (D_2' \cos U_2')$	0·39501
$D_2' . \cos U_2'$ $+ X_2$	2·48319 −1·17652
$D_2' . \cos U_2' + X_2$ $- d_2'$	1·30667 1·50000
X_3	−0·19333
$\log \sin (U_2 + I_2)$ $+ \log r_2$	9·72096n 0·89598n
$\log Y_2$	0·61694
Y_2	4·13943
$\log D_2'$ $+ \log \sin U_2'$	0·39573 8·75891
$\log D_2' . \sin U_2'$	9·15464
$(-) D_2' . \sin U_2'$ $+ Y_2$	0·14277 4·13943
Y_3	3·99666
$2 \log Y_3$ $- \log 2 X_3$	1·20340 9·58733n
$\log \dfrac{(Y_3)^2}{2X_3}$	1·61607n
$\dfrac{(Y_3)^2}{2X_3}$	−41·311

$$r_3 = \frac{(Y_3)^2}{2X_3} + \tfrac{1}{2}X_3 = -41{\cdot}311 - 0{\cdot}097 = -41{\cdot}41 \text{ mm.}$$

Medium-power Microscope Objectives (Lister type)

When dealing with the design of the low-power microscope lens in the previous section, it was evident that a two-lens achromat could be made to be free from spherical aberration for a limited numerical aperture. The corresponding object and image points were on opposite sides of the lens in this case, but it can be shown that with such a lens there is another pair of aplanatic* points, namely when object and image points are on the *same* side of the lens; i.e., when the image is a virtual one (see Fig. 70*a*).

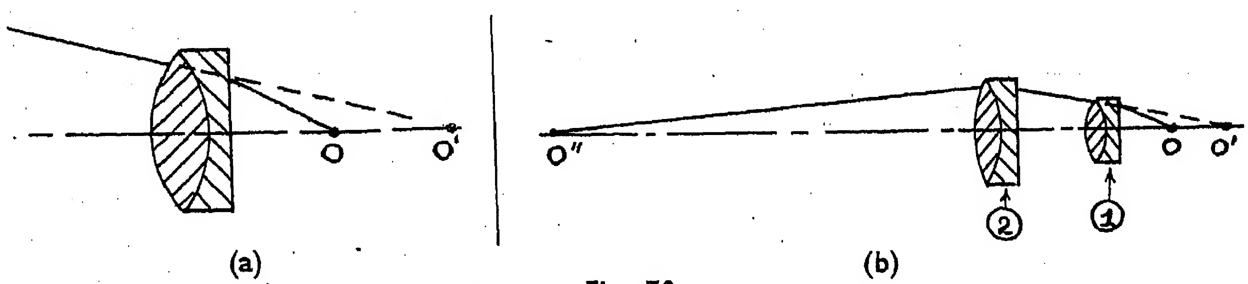


Fig. 70.

Utilizing this property of the aplanatic points J. J. Lister developed a principle for designing higher power microscope objectives by incorporating two achromatic lenses in train, in such a way that the first gave aplanatic refraction with a virtual image at O′ (Fig. 70*b*) this point serving as one of the aplanatic points of the second lens, whilst the conjugate aplanatic point of this lens is at 0″, the position of the final image given by the complete lens system.

As this type of lens is used to a great extent in medium power microscope objectives we cannot do better than take a numerical example to illustrate the way in which the design of this form of lens may be carried out. Let us ask for the following specification :—

Numerical Aperture to be 0·25.

Tube-length to be 160 mm.

Primary magnification × 8·3.

And the glasses :—

	N_D	$\delta N = (N_F - N_C)$	V	$N_D - N_C$	$N_F - N_D$
For the two crown lenses	1·51750	0·00856	60·5	0·00254	0·00602
For the two flint lenses	1·62140	0·01722	36·1	0·00491	0·01231

We must first carry out some minor initial calculations in order to provide the necessary data for the designing work proper.

* "Aplanatic" here refers to freedom from spherical aberration.

Referring to Fig. 71 the Numerical Aperture angle U'_6 is fixed at 0·25 from the specification, and the prescribed magnification as 8·3 times, therefore the initial angle U_1 for the computing work may be obtained as in the previous section from :—

$$\frac{N' \, . \sin U'}{N \, . \sin U} = M, \text{ or } \frac{N' \, . \sin U'_6}{N' \, . \sin U_1} = M$$

i.e.,
$$\frac{0 \cdot 25}{\sin U_1} = -8 \cdot 3 \; ; \quad \therefore \quad \sin U_1 = -0 \cdot 030$$
$$\text{or } U_1 = -1°\text{–}43'\text{–}0''.$$

We now require to know the focal length of each component ; and to do this, first the change in direction of the rays produced by the whole system is required, and secondly the deviation produced by each component individually.

Referring to the first point, we have $u_1 = \sin U_1 = -0 \cdot 03$.

(N.B.—The sign of U_1 is negative according to the convention stated in Chapter 1, and as the Optical Sine Condition is to be fulfilled we must aim at making $u'_6 = \sin U'_6 = 0 \cdot 25$.)

The complete system must therefore produce a change in direction of the paraxial ray of $u'_6 - u_1 = 0 \cdot 25 - (-0 \cdot 03) = 0 \cdot 28$.

Regarding the second point, the question of sub-dividing the amount of " work " done by each of the components comes in, or in other words the allotting of the deviation in the rays produced by each lens.

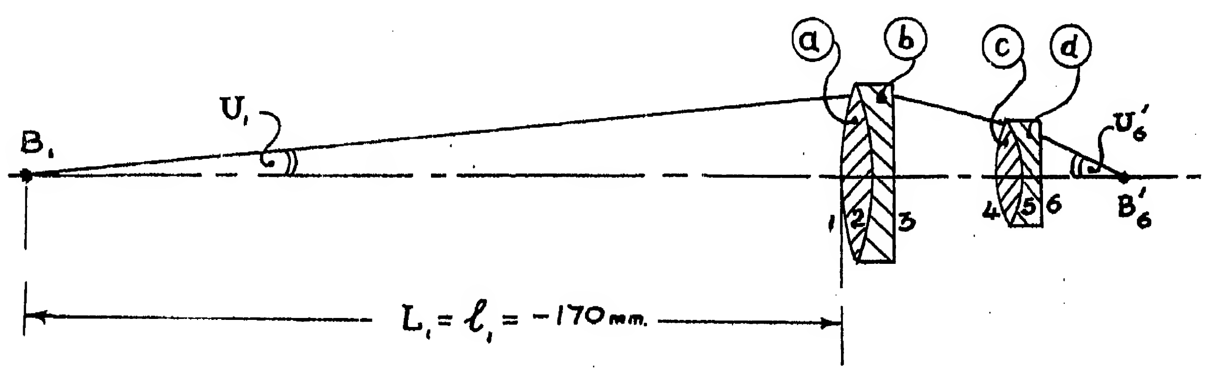


Fig. 71.

A natural choice possibly, would be to distribute the " work " evenly and make each component produce the same amount of *change in direction* of the rays. This is not always the best arrangement of affairs but it will serve quite well to adopt this for the sake of this example.

Consequently, assuming that each component shall do half the " work ", then the back component is to produce a deviation of $\frac{0 \cdot 28}{2}$ or $u'_3 - u_1 = 0 \cdot 14$.

Therefore $u'_3 = 0 \cdot 14 + u_1 = 0 \cdot 14 - 0 \cdot 03 = 0 \cdot 11$.

The required focal length of the back component can be determined from paraxial equations, viz. :—

For by paraxial formulæ $l_1 u_1 = l'_3 u'_3$ or $l'_3 = l_1 \cdot \frac{u_1}{u'_3} = -170 \times \left(\frac{-0 \cdot 03}{0 \cdot 11}\right)$

$$\therefore \quad l'_3 = 46 \cdot 4 \text{ mm.}$$

and as $\frac{1}{l'_3} = \frac{1}{f} + \frac{1}{l_1}$; $\frac{1}{f} = \frac{1}{46 \cdot 4} + \frac{1}{170} = 0 \cdot 0274$

$$\therefore \quad f \text{ (Back component)} = 36 \cdot 5 \text{ mm.}$$

Still dealing with the back component, we now apply the achromatism formula and find the power or total curvature $\mathcal{R}_a$ and $\mathcal{R}_b$ of each lens a and b of the component, from

$$\mathcal{R}_a = \frac{1}{\text{E.F.L.}} \cdot \frac{1}{(V_a - V_b)} \cdot \frac{1}{\delta N_a} = \frac{1}{36 \cdot 5 \times 24 \cdot 4 \times 0 \cdot 00856} = +0 \cdot 1313$$

$$\mathcal{R}_b = \frac{1}{\text{E.F.L.}} \cdot \frac{1}{(V_b - V_a)} \cdot \frac{1}{\delta N_b} = \frac{1}{36 \cdot 5 \times -24 \cdot 4 \times 0 \cdot 01722} = -0 \cdot 0652.$$

Four bendings or shapes of this component are then chosen at equal intervals of R_1 corresponding to (say) $0 \cdot 00$, $0 \cdot 02$, $0 \cdot 04$ and $0 \cdot 06$; and from $\mathcal{R}_a = R_1 - R_2$ and $\mathcal{R}_b = R_2 - R_3$ and taking reciprocals we find the following radii for the various shapes :—

$R_1 =$	$0 \cdot 00$	$0 \cdot 02$	$0 \cdot 04$	$0 \cdot 06$
$r_1 =$	∞	$50 \cdot 00$mm.	$25 \cdot 00$mm.	$16 \cdot 67$mm.
$r_2 =$	$-\ 7 \cdot 62$mm.	$-\ 8 \cdot 98$mm.	$-10 \cdot 95$mm.	$-14 \cdot 03$mm.
r_3	$-15 \cdot 13$mm.	$-21 \cdot 69$mm.	$-38 \cdot 31$mm.	$-163 \cdot 93$mm.

N.B.—The radius r_3 in each case is not yet corrected for achromatism.

Suitable axial thicknesses (based on a 5/1 scale drawing) will be for the crown lens $2 \cdot 7$ mm. and for the flint lens $1 \cdot 5$ mm.

The trigonometrical ray-tracing can now be commenced for each of the four shapes, in order to find the spherical aberration and OSC in each case; this involves tracing marginal and paraxial rays in " brightest light " (Ny) and correcting the last radius for achromatism by the $(d - D)$ method, starting with :—

$L_1 = l_1 = -170$ mm, $U_1 = u_1 = -1°\text{–}43'\text{–}0''$, and the refractive index Ny is calculated as before from

		Crown	Flint
N_D	=	1·5175	1·6214
$+0\cdot188\,(N_F - N_C)$	=	0·0016	0·0032
Ny	=	1·5191	1·6246

The results of the ray-tracings through the four shapes will be found to give the following values :—

R_1	=	0·00	0·02	0·04	0·06
r_3 (by $d-D$ method)	=	− 14·56mm.	− 20·32mm.	− 35·13mm.	− 156·58mm.
L_3'	=	41·521mm.	43·234mm.	43·671mm.	42·866mm.
l_3'	=	42·394mm.	42·419mm.	42·593mm.	43·097mm.
Sph. Ab. LA_3'	=	+0·873mm.	− 0·816mm.	− 1·078mm.	+ 0·231mm.
log sin U_3'	=	9·09300	9·06786	9·05519	9·05565
log u_3'	=	9·09309	9·07928	9·06387	9·04629
OSC_3'	=	− 0·0212	− 0·0073	+ 0·0050	+ 0·0160

At this stage we now proceed to determine the form of the front component. Its focal length must first be obtained ; this involves the use of a paraxial formula (as before) namely $l_6' \,.\, u_6' = l_4 \,.\, u_4$. From the previous calculation on page 151 last line, $u_3' = u_4 = 0\cdot11$, whilst $u_6' = 0\cdot25$ is the specified numerical aperture, but l_4 is not yet known. It may be obtained by assuming (in the first instance) some arbitrary separation of (say) 17·5 mm. of the two components, which from a mean value of $l_3' = 42\cdot5$ mm. would give $l_4 = 25\cdot0$ mm.

(N.B.—The foregoing values are all of a sufficient approximation for use in " thin " lens formulæ.)

$$\text{Hence } l_6' = \frac{25\cdot0 \times 0\cdot11}{0\cdot25} = 11 \text{ mm., and}$$

$$\frac{1}{f_{\binom{\text{front}}{\text{component}}}} = \frac{1}{11} - \frac{1}{25} = 0\cdot0509\text{ ; therefore } f_{\binom{\text{front}}{\text{component}}} = 19\cdot6(4)\text{ mm.}$$

The power or curvature of each lens of the front component is obtained from :

$$R_c = \frac{1}{19\cdot6 \times 24\cdot4 \times 0\cdot00856} = +\,0\cdot2437.$$

and

$$R_d = \frac{1}{19\cdot6 \times -24\cdot4 \times 0\cdot01722} = -\,0\cdot1212.$$

We could then bend this front component into various shapes and continue to trace the rays on from one shape of the back component and find the spherical aberration and OSC in each case ; the procedure would have to be repeated for each other shape of the back component in turn, but obviously this would involve an enormous amount of work. A better way is to carry out ray-tracings from *right-to-left* through the front component and to compare the L_4, U_4, l_4 and u_4 of the rays as they leave surface No. 4 (travelling *left*) with the L'_3, U'_3, l'_3 and u'_3 values of the rays leaving surface No. 3 (travelling to the *right*) ; for instance in Fig. 72*a* U'_3 and U_4 have different inclinations to one another, whilst in Fig. 72*b* the case is illustrated when U'_3 equals U_4 but the two rays do not coincide. The ideal condition (shown in Fig. 71) is that in which any ray traced (left-to-right) from B_1 through the back component shall be *absolutely coincident* with the corresponding ray traced (right-to-left) from B'_6 (conditional with the Optical Sine Relation) in the intervening space between the two components. This principle is sometimes known as the Matching Principle. Failure in perfect matching of the rays is a sure indication of imperfect correction of the complete system, and therefore the application of this principle to microscope lens design is of considerable help.

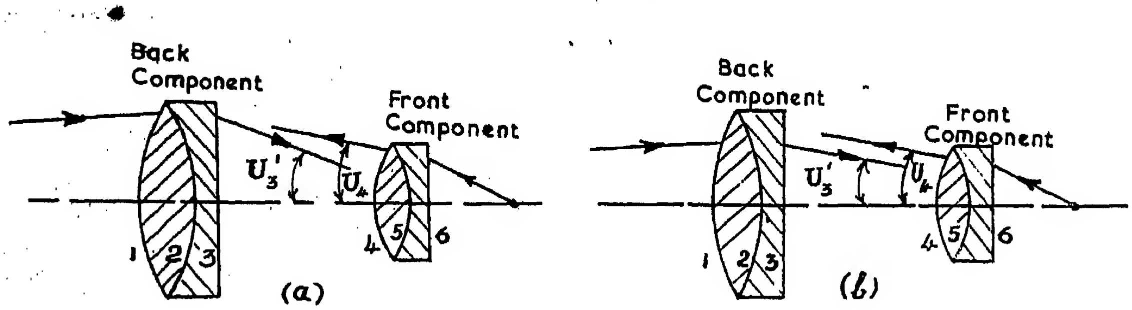


Fig. 72.

Returning to our designing, therefore, and referring to the table on page 153 it will be noted that the log sin U'_3 values are not the same for each of the bendings ; so that in order to apply the "matching principle" it is desirable to have *one* definite inclination of U'_3 to enter the front component.

But it is not necessary to find this by laborious trigonometrical trials, for we can adopt a standard value of U'_3 equal to say 6°–40′–0″ (i.e., log sin U'_3 $= 9\cdot06481$) and then adjust the LA' and OSC' values in the four columns of this table. For we know already that both LA' and OSC' vary, in first approximation, with the square of the aperture and therefore with the square of sin U'_3. We can thus convert the LA' and OSC' values given in the table on page 153 by multiplying them with the square of the ratio : $\dfrac{\text{adopted sine}}{\text{computed sine}}$.

For example, in the first column of the table on page 153 the logarithm of this

ratio will be $9 \cdot 06481 - 9 \cdot 09300 = 9 \cdot 97180$, and the log of the squared ratio will be twice this, namely $9 \cdot 94360$. By adding this to the logs of the LA' and OSC' already found, we obtain the adjusted LA' as $+0 \cdot 767$ and the adjusted OSC' as $-0 \cdot 0187$.

In this way we obtain a new table of values for LA' and OSC' for the uniform value of $U'_3 = 6°–40'–0''$.

For R_1	=	0·00	0·02	0·04	0·06
corrected LA'	=	+0·767 mm.	−0·805	−1·127	+0·241
corrected OSC'	=	−0·0187	−0·0072	+0·0052	+0·0168

These figures can then be plotted as shown in the left half of Fig. 73.

Continuing now with the front component four bendings or shapes may be chosen as follows :—

$R_4 =$	0·04	0·08	0·12	0·16	d'
$r_4 =$	25·00 mm.	12·50 mm.	8·33 mm.	6·25 mm.	
					← 3·0 mm.
$r_5 =$	− 4·91 mm.	− 6·11 mm.	− 8·08 mm.	− 11·95 mm.	
					← 1·0 mm.
$r_6 =$	− 12·12 mm.	− 23·53 mm.	− 400·00 mm.	+ 26·67 mm.	

(N.B.—The radius r_6 in each case is not yet corrected for achromatism and d' (axial thickness is obtained from a scale drawing.)

The marginal ray from the back component at $U_4 = 6°–40'–0''$ to the axis and with $L_4 = 25 \cdot 00$ mm. is then traced on through the above four selected bendings, correcting the last radius for achromatism in each case by the $(d - D)$ method. The following results will be found :—

R_4	=	0·04	0·08	0·12	0·16
r_6	=	− 10·45 mm.	− 21·15 mm.	+ 105·49 mm.	+ 12·83
U'_6	=	15°–42'–49''	14°–56'–25''	14°–53'– 2''	13°–54'–56''
L'_6	=	8·812 mm.	8·818 mm.	8·340 mm.	8·368

r_6 is corrected last surface.

In order to determine the spherical aberration (LA') we must now trace a paraxial ray backwards (i.e., right-to-left) from the point B'_6 (Fig. 71) using sin U'_6 above as the initial u'_6 value and the L'_6 as the initial l'_6 value. The l_4 and u_4 at which this paraxial ray leaves surface No. 4 can then be compared with the L_4 and U_4 of the corresponding marginal ray, and the LA and OSC

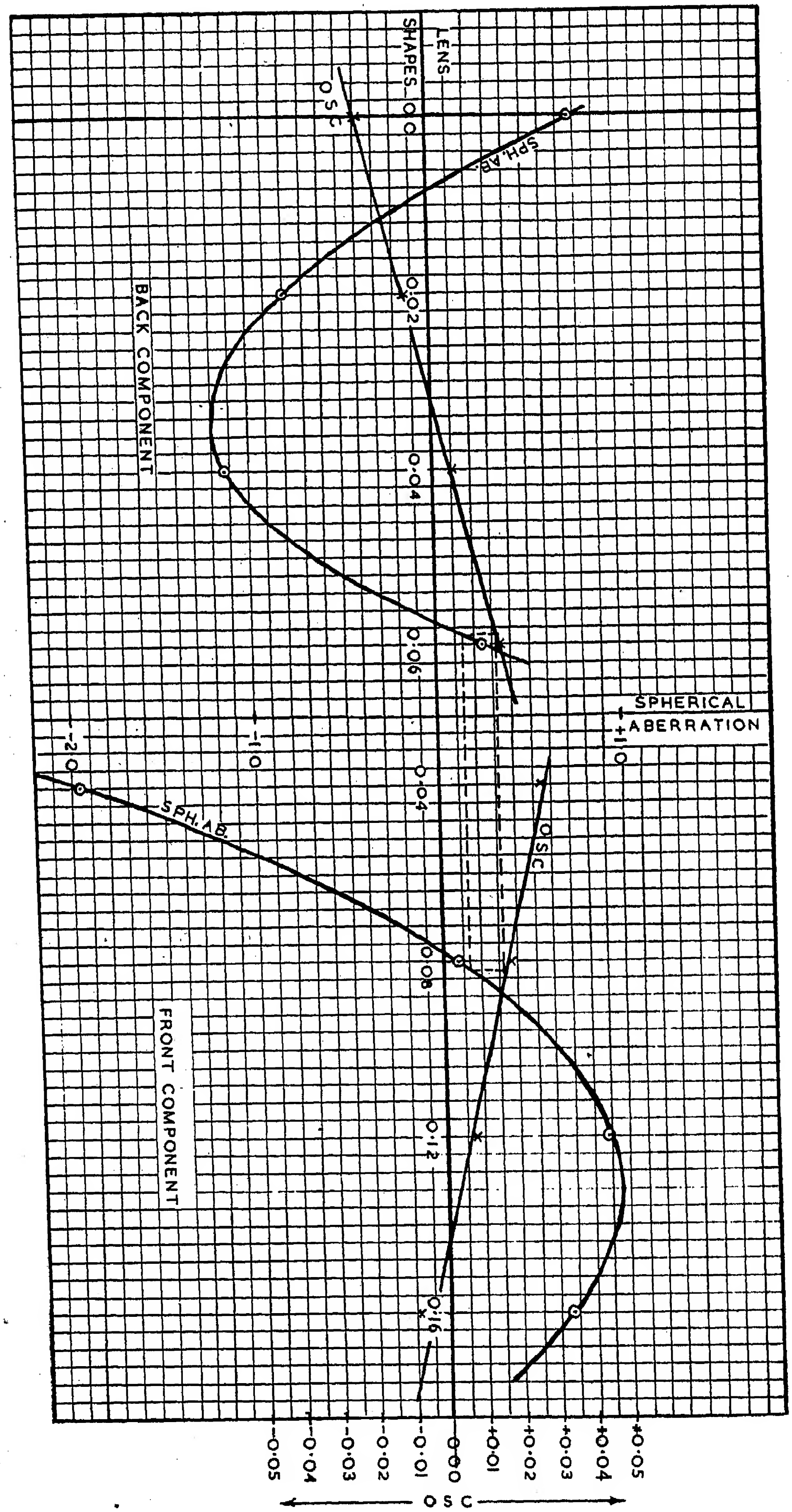

LENS
SHAPES
O·O
OSC
SPH. AB.
BACK COMPONENT
0·02
0·04
0·06
SPHERICAL
ABERRATION
+1·0
−1·0
−2·0
0·04
OSC
SPH. AB.
0·08
FRONT COMPONENT
0·12
0·16
−0·05
−0·04
−0·03
−0·02
−0·01
0·00
+0·01
+0·02
+0·03
+0·04
+0·05
OSC

Fig. 73.

values of the four bendings can thus be found for the front component ; giving the following results :—

R_4	=	0·04	0·08	0·12	0·16
LA_4	=	−1·963 mm.	+0·066 mm.	+0·884 mm.	+0·672
OSC_4	=	+0·0273	+0·0185	+0·0081	−0·0089

The above figures are then plotted on the same graph as those already plotted for the back component but on the right-hand side (see Fig. 73) using curvatures R_4 as abscissa and spherical aberration and OSC as ordinates.

In accordance with the " matching principle " we now have to pick out corresponding bendings or shapes of the two components such that $LA_3' = LA_4$ and *simultaneously* $OSC_3' = OSC_4$; for in shapes so co-ordinated, coincidence of the marginal intersection point B_{M3}' with B_{M4} will also imply coincidence of the paraxial intersection points B_3' and B_4 and therefore spherical correction of the complete objective. Also the equality of $OSC_3' = OSC_4$ will ensure that the corresponding marginal and paraxial rays will have the same direction (i.e., u and U values) in the space between the components and will therefore coincide along their whole length

Evidently these conditions are satisfied by those pairs of bendings or shapes which allow of the fitting in of a perfect rectangle with horizontal and vertical sides between the four aberration curves on Fig. 73.

The easiest way of finding these " matching rectangles " is to slide a ruler over the diagram always keeping it parallel to the abscissa, and to note the vertical distance between the respective LA and OSC curves at each cut of the ruler with (say) the two LA curves.

In this particular case, a matching rectangle will be found (see Fig. 73) where $R_1 = +0·0597$ and $R_4 = +0·0840$. (Sometimes more than one such rectangle may be found.)

Taking, therefore, the above solution for R_1 and R_4 we can find the best shapes of the lenses

from $\begin{cases} \mathcal{R}_a = R_1 - R_2 \\ \mathcal{R}_b = R_2 - R_3 \end{cases}$ and $\begin{cases} \mathcal{R}_c = R_4 - R_5 \\ \mathcal{R}_d = R_5 - R_6 \end{cases}$

and from page 152 $\begin{cases} \mathcal{R}_a = +0·1313 \\ \mathcal{R}_b = -0·0652 \end{cases}$ $\begin{cases} \mathcal{R}_c = +0·2437 \\ \mathcal{R}_d = -0·1212 \end{cases}$

Hence

$r_1 = +16·75$ mm.	$r_4 = +11·90$ mm.
$d_1' = 2·7$ mm.	$d_4' = 3·0$ mm.
$r_2 = -13·97$	$r_5 = -6·26$ mm.
$d_2' = 1·5$ mm.	$d_5' = 1·0$ mm.
$r_3 = -156·25$	$r_6 = -25·97$ mm.

(N.B.—The radii r_3 and r_6 to be corrected for achromatism.)

The final trigonometrical test must now be carried out. A marginal and paraxial ray in *Ny* light is traced (with initial $l_1 = L_1 = -170$ mm. and $u_1 = \sin U_1 = -0 \cdot 0300$) through the entire system, correcting surfaces No. 3 and No. 6 by the $(d-D)$ method for complete achromatism.

After tracing through surfaces No. 1 and 2, the corrected value for r_3 will be found to be $-159 \cdot 5$ mm., and the results after the rays have passed through this surface are as follows :—

$l'_3 = 43 \cdot 525$ mm. $\qquad u'_3 = 0 \cdot 109870$

$L'_3 = 43 \cdot 330$ mm. $\qquad \log \sin U'_3 = 6° - 26' - 18''$

Going on to the second component, we require the air-space d'_3 which is equal to $(L'_3 - L'_4)$ and as L_4 was already decided on as $25 \cdot 00$ mm., $d'_3 = 43 \cdot 330 - 25 \cdot 000 = 18 \cdot 330$ mm. So that $l_4 = 43 \cdot 525 - 18 \cdot 330$. $= 25 \cdot 195$ mm.

Thus the data for the ray-tracing through the front component is :—

$l_4 = +25 \cdot 195$ mm. $\qquad u_4 = 0 \cdot 109870$

$L_4 = +25 \cdot 000$ mm. $\qquad U_4 = 6° - 26' - 18''$

When r_6 is corrected for achromatism by the $(d-D)$ method it will be found to be $-24 \cdot 30$ mm.; and the final figures are :—

$l'_6 = 8 \cdot 848$ mm. $\qquad u'_6 = 0 \cdot 248061$

$L'_6 = 8 \cdot 848$ mm. $\qquad U'_6 = 14° - 18' - 5''$

Thus, the spherical aberration (marginal) $l'_6 - L'_6 = -0 \cdot 000$ mm.

Sph. Ab. Tolerance $= \pm 0 \cdot 036$ mm.

and, the Offence against the Sine Condition $= -0 \cdot 0042$.

OSC Tolerance $= \pm 0 \cdot 0025$.

The zonal spherical aberration has yet to be tested and further it is advisable to check the chromatic correction by tracing C and F rays through the complete lens system at $\sqrt{0 \cdot 5}$ of the semi-aperture. If this is done, it will be found that L'_6 (zonal ray) $= 8 \cdot 810$ mm. giving a zonal spherical aberration value $(l'_6 - L'_{6z})$ of $+0 \cdot 038$ mm. As the marginal spherical aberration has been completely corrected, the full zonal tolerance may be applied, namely $\dfrac{6\lambda}{N' \, . \sin^2 U'_M}$ which is $\pm 0 \cdot 054$ mm.; thus the zonal spherical aberration is within the permissible tolerance. The chromatic aberration trigonometrical test results in a value $(L'_C - L'_F)$ of $+0 \cdot 0016$, whilst its tolerance is $\pm 0 \cdot 0092$.

It is evident, therefore, that this design (apart from the rather high *OSC* value) is quite a good one. Suitable changes to remove the residual *OSC* can be inferred from the graphs.

Effect of cover-glass

Most microscope objectives are designed for use with a cover-glass (of a specified thickness) placed over the specimen; unless they are intended for metallurgical work or for observation on opaque objects.

The introduction of the cover-glass does not affect the aberrations of the lens seriously as far as low-power objectives are concerned, but in higher powers (such as the 8 mm. or 4 mm. lenses) the effect is much more marked and consequently the cover-glass thickness must always be taken into account in the computing work.

It will be of interest (even here) in the case of the 16 mm. objective we have just designed to see how the aberrations are affected. All we need to do is to trace the final rays through a plane surface into a medium of refractive index $N = 1 \cdot 5175$. As the standardized coverglass thickness may be taken as $0 \cdot 17$ mm., the initial values for this ray-trace will be :—

$$\left.\begin{aligned} l'_6 &= 8 \cdot 848 - 0 \cdot 17 = 8 \cdot 678 \text{ mm.} \\ L'_6 &= 8 \cdot 848 - 0 \cdot 17 = 8 \cdot 678 \text{ mm.} \end{aligned}\right\} \text{ and } \left\{\begin{aligned} u'_6 &= 0 \cdot 248061 \\ U'_6 &= 14^\circ - 18' - 5'' \end{aligned}\right.$$

In this way the new spherical aberration and OSC due to the cover glass may be determined and compared with the values already obtained on page 158.

Higher Power Microscope Objectives

In microscope objectives exceeding a Numerical Aperture of about $0 \cdot 30$ it becomes necessary to use three or even four separated components in the construction of the lens-system in order to secure proper correction of the various aberrations. This is due to the fact that the large angular change in direction of the rays from the object to the image (sometimes as much as 60 degrees) cannot be " handled " by two lenses (such as in the case of the Lister

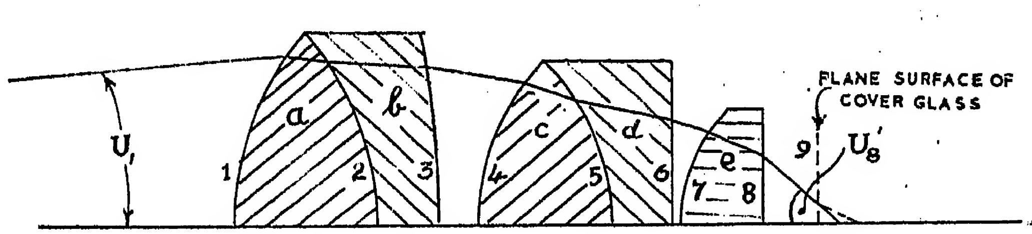


Fig. 74.

objective) without introducing excessively large angles of incidence on the lens surfaces which in turn produce higher order aberrations. It is better, therefore, to employ more lenses and to distribute the amount of " work to be done " more evenly.

The design of a high-power microscope object is a difficult and exacting task; moreover, considerable time and experience is necessary in order to produce a good design. On account of this, only an illustrative example of the various steps to be taken will be given here, leaving the detail for the more advanced worker.

In the highest powers, such as the 2 mm. objectives, the attainment of the desired correction depends almost entirely on empirical trials guided by experience, the latter being most easily acquired by progressive studies commencing with the simple two lens achromatic microscope objective and gradually advancing to the more complex forms.

For the purpose of the introductory work with a higher power microscope objective, let us take one of 4·8 mm. focal length and a Numerical Aperture of 0·575. This will serve to illustrate how the difficulties in designing begin to increase without making these too great by attempting a N.A. of, say, 0·80 (a not unusual value for a nominal 4 mm. objective).

The first thing we must do is to calculate and collect some initial data for the design.

Assuming that we are going to use three components to make up the complete lens-system, then by referring to Fig. 74 we have :—

$$\text{Numerical Aperture} = 0{\cdot}575 = \sin U_8' ; \quad \therefore \quad U_8' = 35^\circ.$$

$$\text{and the Magnification} = \frac{\text{Optical Tube Length}}{f_0} = \frac{160}{4{\cdot}8} = 33\times.$$

$$\text{From } N \,.\, h \,.\, \sin U = N'\,h' \sin U' ; \quad 33 = \frac{\sin U_8'}{\sin U_1} ; \text{ so that}$$

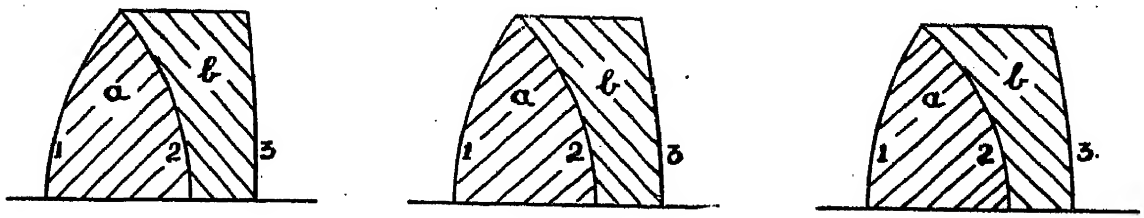
$$\sin U_1 = 0{\cdot}0174, \text{ or } U_1 = 1 \text{ degree (very closely).}$$

Glasses :—The choice of glasses to use requires a certain amount of previous experience, but for the sake of this example we will take the following :—

Lens	N_D	$N_F - N_C$	V	N_C	N_F	N_y
a and c	1·50980	0·00788	64·7	1·50745	1·51533	1·51128
b and d	1·64850	0·01916	33·8	1·64306	1·66222	1·65210
e and cover glass	1·51623	0·00853	60·5	1·51369	1·52222	1·51783

The change in direction of the rays in passing from the object to the image through the whole system is $U_8' - U_1 = 35^\circ - (-1^\circ) = 36^\circ$.

In sub-dividing the "work done", let us assume that each component does one third of the work, from which each has to produce a deviation of



$R_2 = -\ 0\cdot180$
$r_1 = +\ 5\cdot64$ mm.
$r_2 = -\ 5\cdot64$ mm.
$r_3 = -30\cdot21$ mm.

$R_2 = -\ 0\cdot200$
$r_1 = +\ 6\cdot36$ mm.
$r_2 = -\ 5\cdot00$ mm.
$r_3 = -18\cdot83$ mm.

$R_3 = -\ 0\cdot220$
$r_1 = +\ 7\cdot29$ mm.
$r_2 = -\ 4\cdot54$ mm.
$r_3 = -13\cdot68$ mm.

Fig. 75.

12 degrees. Knowing this, the focal lengths of each of the three components may be calculated and will be found to be :—

component $ab = 11\cdot5$ mm. ; component $cd = 7\cdot2$ mm. ; lens* $e = 5\cdot8$ mm.

Taking component ab first, the powers R_a and R_b of the lenses a and b may be determined for the achromatism condition from formula Chr (4) namely :—

$$R_a = \frac{1}{\text{E.F.L. } (V_a - V_b)\delta N_a} = \frac{1}{11\cdot5 \times 30\cdot9 \times 0\cdot00788} = +\,0\cdot3572$$

$$R_b = \frac{1}{\text{E.F.L. } (V_b - V_a)\delta N_b} = \frac{1}{11\cdot5 \times -30\cdot9 \times 0\cdot01916} = -\,0\cdot1469$$

With $R_a = R_1 - R_2$ and $R_b = R_2 - R_3$, various shapes of the lens component ab can then be selected by choosing suitable values for R_2. For example, by making R_2 equal in turn to $-\,0\cdot180$, $-\,0\cdot200$ and $-\,0\cdot220$, the three radii for each shape of lens may be calculated and these shapes are then drawn at a scale of 10 to 1 (see Fig. 75). The axial thickness may also be decided from these drawings, namely $d_1' = 2\cdot60$ mm. and $d_2' = 1\cdot10$ mm. During the process of the design it will be found advisable to take two other "bendings" (such as $R_2 = -\,0\cdot190$ and $-\,0\cdot210$ for example) in order to make sure of the correct shape of the parabola ; for the higher aberrations may cause the primary spherical aberration values to vary to such an extent that the plotting of these points may give an irregular-looking curve.

* In early microscope objectives of this type, the front lens was sometimes an achromatized component ; but it has been shown that the lens e may be of one glass only and the chromatic aberration introduced by it, compensated for by either one or both of the components ab and cd.

The spherical aberration (marginal) and the OSC′ must then be determined trigonometrically for each shape, employing the initial data as $U_1 = u_1 = -1°\text{–}0'\text{–}0''$ and $L_1 = l_1 = -160$ mm. with refractive index Ny for the two glasses. The correction of the third radius for complete achromatism by the $(d-D)$ method must, of course, be carried out in every case.

The values obtained have now to be corrected in the ratio of $\left(\dfrac{\text{adopted sine}}{\text{computed sine}}\right)^2$ in order to obtain the spherical aberration in each case for one definite inclination of U_3', as explained previously on page 154.

The corrected values for both the spherical aberration and the OSC' of this component are then plotted.

We now have to deal with the "front unit", comprising the lens e and the component cd. We first trace incident marginal and paraxial rays (in N_y light) *right-to-left* inside the cover glass* from the object situated at a distance of 0·17 mm. (i.e., the standard cover-glass thickness) and with an initial angle of 22 degrees, also rays in N_C and N_F lights at 15° to the axis. When the marginal ray emerges into air from the plane surface of the cover glass it will make an angle of 35 degrees to the axis (i.e., as calculated in the initial data). The front lens e may now be placed in position using an air space of (say) 0·50 mm. between surface 8 and 9; the shape of this lens has yet to be decided but as its focal length is already known (namely 5·8 mm.) it may be "bent" into any desired form and a quite natural choice for this might be to make surface No. 8 plane, in which r_7 would be $+3{\cdot}00$ mm.

Assuming therefore that this shape is used as a first trial, the ray-tracing is continued on right to left through surfaces No. 8 and No. 7 of the lens e.

The shape of component cd must now be prepared. As its equivalent focal length is known (namely 7·2 mm.) the power or curvature of the lens d may be obtained from :—

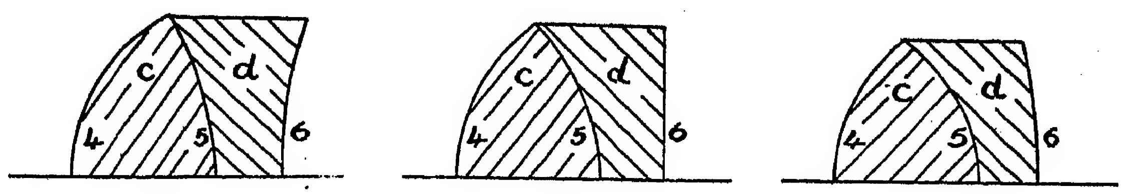
$$\mathcal{R}_d = \frac{1}{\text{E.F.L.} \times (V_d - V_c)\delta N_d} - \left(\frac{\text{Res. Chr. Ab. Term}}{\delta N_d} \cdot \frac{V_c}{V_d - V_c}\right)$$

where the residual chromatic aberration term is that given on page 27. The chromatic aberration $(L_C' - L_F')$ of the front lens e will be found to be $-0{\cdot}0040$ mm.; using this value and the intersection lengths L_C' and L_F', the residual chromatic aberration term may be calculated. Putting the appropriate values to the other terms in the above equation, we find the curvature $\mathcal{R}_d = -0{\cdot}3118$.

As $\mathcal{R}_d = R_5 - R_6$, the shape of this lens may be varied by substituting suitable values for R_5; for example, by putting R_5 equal in turn to, say,

* The cover glass can be treated as an addition of thickness to the front lens if this has a plane surface.

$-0\cdot225$, $-0\cdot300$ and $-0\cdot375$, three shapes may be chosen, and their ponding radii and axial thicknesses drawn to a 10 to 1 scale as shc Fig. 76. (N.B.—There is no need to find the power or curvature of because radius No. 4 has to be adjusted for achromatism and a change will affect the power of lens *c* appreciably.)



$R_5 = -\ 0\cdot225$
$r_6 = +11\cdot52$ mm.
$r_5 = -\ 4\cdot44$ mm.
$r_4 =$ to be corrected for achromatism.

$R_5 = -\ 0\cdot300$
$r_6 = +84\cdot74$ mm.
$r_5 = -\ 3\cdot33$ mm.
$r_4 =$ to be corrected for achromatism.

$R_5 = -\ 0\cdot375$
$r_6 = -15\cdot82$ mm.
$r_5 = -\ 2\cdot67$ mm.
r_4 to be corrected for achromatism.

Fig. 76.

Choosing therefore each of the three shapes (indicated in Fig. 76) in turn, the ray-tracing is continued on right to left from the front lens *e* through surfaces No. 6 and No. 5 and then through surface No. 4 after it has been corrected for achromatism by the $(d-D)$ method.

In this way the front unit as a whole* may be achromatized, but the spherical aberration and the OSC' of this unit will be revealed for the three "bendings" of component *cd*. The aberration values found in this case must again be corrected for one adopted inclination of $U'_3 = U_4$.

These corrected values are then plotted on the same graph mentioned above and the "matching principle" applied as already described on page 154 in the previous section. A matching rectangle may then be drawn, from which we can read off from the abscissa the curvature values of the contact surfaces R_2 and R_5 giving the most suitable solution for the solving of our problem. The resulting specification of the complete lens-system can then be drawn up for trigonometrical testing.

For the final ray-trace, it is advisable to take a marginal, a zonal and paraxial ray (with $U_1 = u_1 = -1°\text{–}0'\text{–}0''$, $U_z = -0°\text{–}42'\text{–}0''$ and $L_1 = l_1 = -160$ mm.) in Ny light; and a zonal ray with $U_z = -0°\text{–}42'\text{–}0''$) in N_C and N_F light in order to test the colour correction. The OSC' must also be tested.

When these various aberration values have been determined, their respective tolerances must be calculated and the results compared. It may be that some of the aberrations are outside the permissible tolerances. It

* Sometimes the chromatic aberration of lens *e* may be compensated by both components *ab* and *cd*, instead of by *cd* alone.

must be remembered, however, that with all complex lens systems and especially those with steeply curved surfaces (such as in this case) higher order aberrations are introduced and they are often so large that the primary aberrations are considerably affected. If adjustment is required in order to reduce one or more of the aberrations, it is interesting to note that the spherical aberration can be varied by a change in shape or thickness (or both) of the lens e; whilst quite large amounts of coma may be corrected by controlling the curved surface of the front lens e, this aberration being unaffected either by the plane surface of this lens or at the cover-glass surface. Thus we see the advantage to be gained by carrying out the ray-tracing in the reverse direction to that in which the lens-system is used in practice, as mentioned earlier in this chapter.

Optical Path Method

As has been mentioned before, the treatment of low numerical aperture microscope objectives may be dealt with by the purely geometrical ray-tracing methods for bringing the marginal, paraxial and 0·7071 zonal rays to a common focus or within their permissible tolerances. These results may be relied on to give definition of the image approximating to the theoretical resolving power of the lens.

When, however, the N.A. is made larger the higher order aberrations introduced may give rise to excessive-looking values for the longitudinal aberrations. These may not necessarily prove detrimental to the final image if considered from the physical aspect of the image.

This whole subject has been discussed in an admirable manner by Conrady in the *Dictionary of Applied Physics*, Vol. 4, pp. 213–228, and its treatment is based on :—

(i) the fact that the image of an object takes the form of a diffraction disc (due to the undulatory theory of light) and not that of a mathematical image point as assumed by the purely geometrical theory.

(ii) the shape of the wave-front as it leaves the lens-system ; and

(iii) the optical path length of the "rays" during their passage from the object to the image via the lens-system.

Readers would benefit considerably by studying the article (referred to above) carefully before applying the method indicated only in abbreviated form in this chapter.

The differences of optical paths are a direct measure of the departure from the true spherical form as the wave-front emerges from the lens-system.

$\frac{1}{2}U$ $-\frac{1}{2}U'$
$\frac{1}{2}(U-U')$
$\frac{1}{2}I$ $-\frac{1}{2}U'$
$\frac{1}{2}(I-U')$
$\log N'$ $+\log PA$ $+\log \sin I'$ $+\log \sin \frac{1}{2}(U-U')$ $+\log \sin \frac{1}{2}(I-U')$
log numerator
$\log 2$ $+\log \cos \frac{U}{2}$ $+\log \cos \frac{I}{2}$ $+\log \cos \frac{I'}{2}$ $+\log \cos \frac{U'}{2}$
log denominator
log numerator – log denominator
$\log OPD$
OPD

(a)

L
U
$\log L$ $+\log \tan U$
$\log Y$
$\frac{1}{2}U$ $+\frac{1}{2}U'$
$\frac{1}{2}(U+U')$
$\frac{1}{2}U$ $-\frac{1}{2}U'$
$\frac{1}{2}(U-U')$
$\log N'$ $+\log Y$ $+\log \sin U'$ $+\log \sin \frac{1}{2}(U+U')$ $+\log \sin \frac{1}{2}(U-U')$
log numerator
$\log 2$ $+2\log \cos \frac{U}{2}$ $+2\log \cos \frac{U'}{2}$
log denominator
log numerator – log denominator
$\log OPD$
OPD

(b)

Fig. 77. Templets for OPD calculations.
(a) for spherical surfaces.
(b) for plane surfaces.

For that reason they lead to the limits within which the particular aberration concerned must be corrected if the full resolving power and defining power of a lens-system is to be realized; these limits permit a considerable increase

in the aperture of a lens system which, judged geometrically, might appear considerably over or under corrected.

Stated briefly, equations can be derived which give the aberrations in terms of differences of optical paths (in wavelengths of light as a unit) and which are used in conjunction with the trigonometrical ray-tracing values. For example, in the case of spherical aberration the following formulæ will give the optical path difference of a marginal ray with respect to a truly axial ray at each surface of the lens-system :—

$$OPD_M = \frac{N' \, . \, PA \, . \sin I' \, . \sin \tfrac{1}{2}(U - U') \sin \tfrac{1}{2}(I - U')}{2\cos \frac{U}{2} \cos \frac{I}{2} \cos \frac{I'}{2} \cos \frac{U'}{2}} \quad \text{for a spherical surface}$$

$$OPD_M = \frac{N' \, . \, Y \, . \sin U' \, . \sin \tfrac{1}{2}(U + U') \sin \tfrac{1}{2}(U - U')}{2\cos^2 \frac{U}{2} \, . \, \cos^2 \frac{U'}{2}} \quad \text{for a plane surface}$$

where $Y = L \tan U = L' \tan U'$.

The working out of these equations is carried out surface by surface and the algebraic sum of the OPD values at each surface is made, this summation giving the final *OPD* at the marginal focus. The arrangement of the calculations when using the above equations are shown in Fig. 77.

The *OPD* method also allows of information regarding correction of the chromatic aberration (see p. 224, *Dict. Appl. Physics*, Vol. 4) but probably the most practical way of designing the higher power microscope objectives is to correct the various components for colour by the $(d - D)$ method, then to correct the spherical aberration by bringing the marginal and paraxial rays to a common focus by trigonometrical methods and then to see that the zonal spherical aberration tested by the *OPD* method comes within its tolerance. Finally the optical sine condition must be satisfied.

Immersion Objectives

Microscope objectives having the highest numerical apertures are generally termed " immersion " objectives, on account of the fact that a fluid (usually cedar wood oil) is used between the front lens of the objective and the top of the cover glass, and as the refractive index of these three is arranged to be the same the object may be treated as being " immersed " in one medium of this uniform refractive index. The purpose of such an arrangement of affairs is to enable the principle of " aplanatic refraction " to be applied to a steeply curved hemi-spherical front lens which thus allows a large solid cone of light to be taken in from the object without introducing spherical aberration into the image formed by this lens.

For example (referring to Fig. 78), the full line circle represents a s of glass having a refractive index N and radius r. If an object O is situ at a distance from C equal to $\frac{r}{N}$, all rays (of one colour) from O after refracti at the surface of the sphere into air will appear to come from a virtual image O′ whose distance from C will be $r \,.\, N$ and there will be no spherical aberration present. In practice, of course, the object cannot be mounted *inside* the glass sphere, so the latter may be cut in half and a fluid of the same refractive index as the hemi-sphere employed to complete the condition illustrated in the figure thus enabling the object to be placed at O.

This is a useful application of the aplanatic refraction principle for it allows rays from the object making considerable angles (e.g., between 50

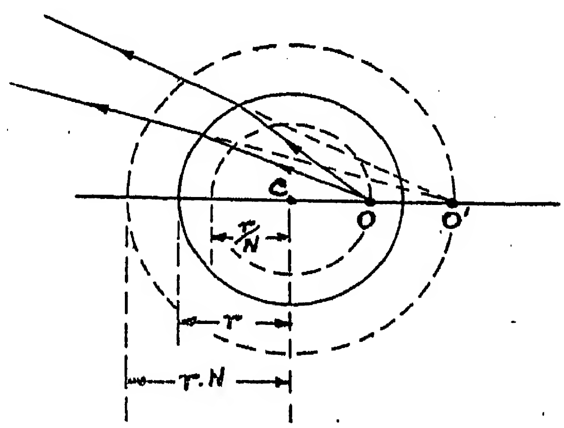


Fig. 78.

and 60 degrees) with the axis to be "bent round", free from spherical aberration, to an inclination that can be dealt with by lens components which follow (see Fig. 79). Frequently it is necessary to introduce an additional lens (of meniscus shape) behind the hemi-sphere in order to "bend" the marginal rays still further round before they enter the achromatizing components.

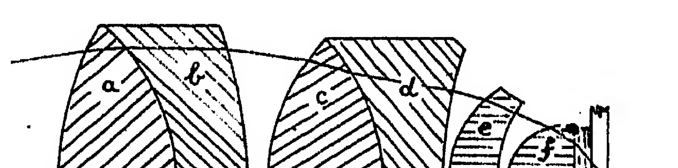


Fig. 79.

When commencing the design of an immersion objective it is advisable and extremely helpful to draw out the front unit to a scale of about 100 to 1,

and to fix some of the initial data by purely graphical methods. For example (see Fig. 80) let us set off the cover glass thickness equal to 17 mm. (i.e., 100 times the standard cover-glass thickness) and an oil film of 21·1 mm. (in reality 0·210 mm.). This latter value is obtained by exact calculation (see later).

Assuming a refractive index for the cover glass, the immersion oil, and the front lens, as 1·51690; then a numerical aperture of (say) $1\cdot 24 = N \,.\, \sin U$ would provide an angle U_{10} of 55 degrees. This angle may now be drawn from the object, and will thus help in deciding both the diameter and the radius of the hemi-spherical front lens.*

From the drawing we find that we can make the radius of the hyper-hemisphere equal to 0·617 mm. and its axial thickness equal to 0·643 mm.; this will take in the extreme marginal ray with initial angle $U = 55$ degrees.

Having decided on the radius r_9, the distance L_9 of the object from the pole of the surface A_9 can now be determined by the aplanatic refraction principle, namely $L_9 = r + \frac{r}{N}$ whence $L_9 = 0\cdot 617 + \frac{0\cdot 617}{1\cdot 5169} = 1\cdot 023$ mm.

The marginal ray can then be traced at this stage (by graphical means) into air and it will be found that the angle which the emergent ray makes with the axis is approximately 33 degrees. This is too great an angle to expect the two following achromatic components to deal with without producing excessive higher order aberrations; it is therefore desirable to introduce

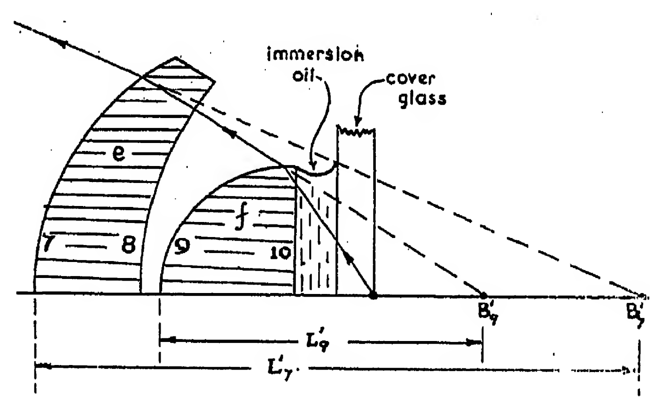


Fig. 80.

another lens *e* (Fig. 80) to enable this angle of 33 degrees to be reduced. We can again apply the principle of aplanatic refraction to this lens in order to remove spherical aberration, and its shape is thus automatically decided;

* From a practical point of view the front lens is sometimes made slightly greater than a hemisphere; this enables greater accuracy in the determination of the radius, for measurements can then be made across the equator of the sphere.

for surface No. 8 can be made of such a radius that the incident ray strikes the surface normally, and surface No. 7 the radius of a sphere which will comply with the condition that the distance between the intersection point B'_9 and the centre of the sphere shall be $\dfrac{r}{N}$.

In order to fix r_7, however, it is necessary to decide on the position of the pole of the surface A_7 which may be done on the drawing board to give suitable axial thickness and diameter of lens *e*. A suitable air space between the lenses will be 0·10 mm., and the axial thickness of lens *e*, 0·510 mm.

The position of B'_9 should, however, now be determined by exact calculation. Its distance from the centre C of the sphere is equal to $N \,.\, r = 0\cdot937$ mm.; this distance $+0\cdot617 + 0\cdot100 + 0\cdot510$ will give the exact distance of B'_9 from A_7, namely 2·164 mm.

Having thus determined the distance between B'_9 and A_7, this numerical value will be equal to $\dfrac{r_7}{N} + r_7$; hence $\dfrac{r_7}{N} + r_7 = 2\cdot164$ from which r_7 may be found, namely 1·305 mm.

The first part of the procedure in the systematic design of the immersion objective is to determine the amount of chromatic aberration introduced by the two front lenses, as this has to be compensated for by the components following in the optical train. To do this we must trace a zonal ray from the object point B_{10} right to left in C and F light; it will be useful at the same time to trace an extreme marginal and a paraxial ray in *Ny*-light, in order to use these later on in the calculations and also to check the elimination of spherical aberration by the utilization of the "aplanatic refraction" principle.

The initial data for the ray-tracing will be :—$L_M = L_Z = l =$ distance between A_9 and $B_{10} = +1\cdot023$ mm.

$$U_M = +55°\text{–}0'\text{–}0'' = \text{nominal } u\,; \quad U_Z = +35°\text{–}0'\text{–}0''.$$

The optical constants for the cover-glass, the immersion oil and the hyper-hemisphere may be taken as follows :—

N_D	$N_F - N_C$	V	$N_D - N_C$	$N_F - N_D$
1·51690	0·00853	60·6	0·00261	0·00592

Tracing through surface No. 9, we find

$L'_C = +1\cdot547$ mm.	$U'_C = +22° - 18' - 25''$	$L'_M = +1\cdot554$ mm.	$U'_M = +32° - 39' - 12''$
$L'_F = +1\cdot562$ mm.	$U'_F = +22° - 4' - 54''$	$l' = +1\cdot554$ mm.	$u' = +0\cdot539495$
		$L'_Z = +1\cdot554$ mm.	$U'_Z = 22° - 11' - 45''$

Thus the chromatic aberration is $-0 \cdot 015$ mm., but the intersection lengths in the spherical aberration test indicate the validity of the aplanatic refraction principle.

As the marginal angle is as large as $32\frac{1}{2}$ degrees it is advisable to introduce a further lens in order to reduce this angle. Assuming a separation of $0 \cdot 1$ mm. between lens f and lens e, the radius of surface No. 8 (in order that the rays shall make normal incidence with this surface) must be $1 \cdot 554 + 0 \cdot 100 = 1 \cdot 654$ mm.

As the marginal, zonal and paraxial rays in *Ny*-light strike this surface normally there is no need, of course, to do any computation at this surface for these rays; but the C and F rays do *not* strike the surface normally and therefore they must be traced through surface No. 8. All five rays should then be traced through surface No. 7 of radius $1 \cdot 305$ mm.

The results after surface No. 7 will be found to be :—

$L'_C = +\ 3 \cdot 251$ mm.	$U' = +\ 14° - 34' - 56''$	$L'_M = +\ 3 \cdot 282$ mm.	$U'_M = +\ 20° - 50' - 21''$
$L_F = +\ 3 \cdot 315$ mm.	$U'_F = +\ 14° - 15' - 17''$	$l' = +\ 3 \cdot 282$ mm.	$u' = +\ 0 \cdot 355604$
		$L'_Z = +\ 3 \cdot 282$ mm.	$U'_Z = +\ 14° - 25' - 14''$

Thus the chromatic aberration introduced by the two front lenses (or duplex front as it is sometimes called) is $-0 \cdot 064$ mm., and the spherical aberration is still zero.

It will be noticed that the marginal ray emerges at about 21 degrees* with the axis and the change in direction now needed to bring this ray to the final image plane is therefore approximately 22 degrees. This can be dealt with by the two following achromatic components without much difficulty.

The procedure from now on is similar to that already shown in the case of the 5 mm. lens design given on pages 160 to 164. Knowing the change in direction (i.e., 22 degrees) the focal lengths of the components *cd* and *ab* (Fig. 79) can be determined and hence their total power or curvature, provided the glasses to be used are decided on. This decision again depends to some extent on experience, but it is generally advisable to use pairs of glasses which have a large difference of *V*-values, such as for example a hard crown combined with an extra dense flint or perhaps fluorite combined with a dense flint.

Having chosen a number of shapes of component *cd** the rays from the duplex front are traced on right to left and surface No. 4 adjusted for achromatism in each case; the spherical aberration will also be obtainable in each case. The latter together with the *OSC* values may then be plotted against the curvature of the contact surface of component *cd*. Similar values when determined trigonometrically by left to right ray-tracing through component

* The inclination of 21 degrees is of the same order as that obtained for the trigonometrically traced emerging ray from the front lens of the 5 mm. objective in the previous section.

*ab** can also be plotted on the same graph; and the matching principle applied as indicated previously on two occasions in this chapter.

In this way, the best solution may be obtained, and the whole system should then be tested by trigonometrical and *OPD* methods as already explained.

It is not necessary to give here numerical details of the latter part of the work in this design, for the methods have already been outlined in the previous pages. The main point of this section is to illustrate the initial stages in the design of an immersion objective.

* The chromatic correction of the lenses *e* and *f* may be distributed between the two components *cd* and *ab* instead of by *cd* alone.

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